Maximum Transition Run Codes for Data Storage Systems

Jaekyun Moon and Barrett Brickner
Center for Micromagnetics and Information Technologies
Department of Electrical Engineering, University of Minnesota
Minneapolis, Minnesota 55455

Abstract – A new code is presented which improves the minimum distance properties of sequence detectors operating at high linear densities. This code, which is called the maximum transition run code, eliminates data patterns producing three or more consecutive transitions while imposing the usual k-constraint necessary for timing recovery. The code possesses the similar distance-gaining property of the (1,*k*) code, but can be implemented with considerably higher rates. Bit error rate simulations on fixed delay tree search with decision feedback and high order partial response maximum likelihood detectors confirm large coding gains over the conventional (0,*k*) code.

I. INTRODUCTION

In this paper, we present a new code designed to improve the distance properties of sequence detectors operating at relatively high linear densities. The basic idea is to eliminate certain input bit patterns that would cause most errors in sequence detectors. More specifically, the code eliminates input patterns that contain three or more consecutive transitions in the corresponding current waveform, and, as a result, the performance of any near-optimal sequence detector improves substantially at high linear densities [1][2]. This code constraint, designated the maximum transition-run (MTR) constraint, can be realized with simple fixed-length block codes with rates only slightly lower than the conventional (0,*k*) code. Bit error rate (BER) simulation results with fixed delay tree search with decision feedback (FDTS/DF) detection and high order partial response maximum likelihood (PRML) detection confirm a large coding gain of the MTR codes over the conventional (0,*k*) code.

II. CODING METHODS

Investigation of high density error patterns in FDTS/DF detection reveals that errors arise mostly due to the detector's inability to distinguish the minimum distance transition patterns, four pairs of which are shown in Fig. 1. These pairs of magnetization waveforms give rise to an NRZ input error pattern of $e_k=\pm\{2\ -2\ 2\}$, assuming input data take on +1's and -1's. The proposed approach is to remove data patterns allowing this type of error pattern through coding. The potential improvement in the FDTS detection performance using this approach can be estimated by computing the increase in the minimum distance between two diverging lookahead tree paths after removing the paths that allow the $\pm\{2\ -2\ 2\}$ error events [3]. A simple minimum distance analysis for PRML systems reveals that this is also a critical error pattern in high order PRML systems such as E^2PR4ML. Note that a traditional (1,*k*) runlength limited (RLL) code eliminates all eight transition patterns shown in Fig. 1 [4][5], but the rate penalty is typically too large to see any coding gain unless the linear density is very high. The idea of MTR coding is to eliminate three or more consecutive transitions, but allow the dibit pattern in the written magnetization waveform. Thus, with MTR coding, the error events of the form $\pm\{2\ -2\ 2\}$ will still be prevented as with (1,*k*) coding, but the rate penalty is significantly smaller than that of the typical (1,*k*) RLL code. Notice that with the MTR constraint, the write precompensation efforts can be directed mainly on dibit transitions, unlike in conventional (0,*k*) coded systems. An independent study also suggests that removing long runs of consecutive transitions improves the offtrack performance in some PRML systems [6]. There exist other types of code constraints that can offer similar distance-enhancing properties for high order PRML systems [7].

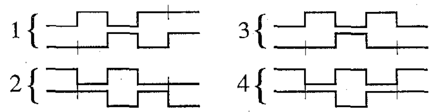

Fig. 1: Pairs of write patterns causing most errors in sequence detection at high linear densities.

Fig. 2 shows the state diagram of the MTR code based on the NRZI convention, where 1 and 0 represent the presence and absence, respectively, of a magnetic transition. Also included is the usual *k*-constraint for timing recovery. The capacity of the code can be obtained by finding the largest eigenvalue of the adjacency matrix for the given state diagram [8]. The capacities for different *k* values are given in Table 1.

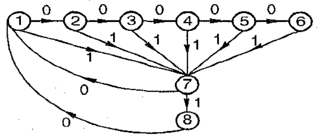

Fig. 2: State transition diagram for the MTR code with *k*=6.

k	capacity	k	capacity
4	.8376	8	.8760
5	.8579	9	.8774
6	.8680	10	.8782
7	.8732	∞	.8791

Table 1: Capacities for MTR codes.

While state-dependent encoders and sliding-block decoders can be designed for the MTR constraint (which can be easily generalized to limit any runs of consecutive transitions), we observe that simple fixed-length block codes can be realized with

Manuscript received March 4, 1996. This work was supported in part by Seagate Technology and the National Storage Industry Consortium (NSIC).

0018-9464/96$05.00 © 1996 IEEE

good rates and reasonable k values. A computer search is utilized to first find all n-bit codewords that are free of an NRZI 111 string or k+1 consecutive NRZI 0's. Then, in order to meet the MTR constraint at the codeword boundaries, words that start or end with an NRZI 11 string are removed. Also, the k constraint is satisfied at the boundary by removing the words with k_1+1 leading 0's or k_2+1 trailing 0's, where $k_1+k_2=k$. Finally, if the number of the remaining codewords is greater than or equal to 2^m, then those codewords can be used to implement a rate m/n block code. Table 2 shows important code parameters for representative block codes obtained through computer search. The efficiency was found by dividing the code rate m/n by the capacity computed for the given value of k and the MTR constraint. As an example of an MTR block code, 16 codewords required to implement the rate 4/5 code with k=8 are given in Table 3.

m	n	k	eff.	No. avail. codewords	No. needed codewords
4	5	8	.91	16	16
8	10	6	.92	282	256
9	11	6	.94	514	512
10	12	8	.95	1,066	1,024
14	17	6	.95	18,996	16,384
16	19	7	.96	69,534	65,536
24	28	8	.98	17,650,478	16,777,216

Table 2: Parameters for MTR block codes.

00001	00110	01100	10010
00010	01000	01101	10100
00100	01001	10000	10101
00101	01010	10001	10110

Table 3: A rate 4/5 MTR block code with k=8.

III. MODIFIED DETECTION AND DISTANCE INCREASE

To realize the coding gain at the detector output, the detector has to be modified. In the case of PRML systems, this amounts to removing those states and state transitions that correspond to the illegal data patterns from the trellis diagram. For the FDTS/DF detector, the code-violating lookahead paths must be prevented from being chosen as the most-likely path, a technique similar to the one used in the (1,7) coded FDTS/DF channel [9]. To illustrate the idea, consider Fig. 3 that shows a τ=2 lookahead tree utilized in FDTS/DF detection. By utilizing the past decision, an illegal path, which contains three consecutive transitions, can be identified as indicated by either the solid (when the past decision is -1) path or the shaded (when the past decision is 1) path. The complexity of the FDTS/DF detector can also be reduced considerably with the MTR code, as elaborated in a companion paper [10].

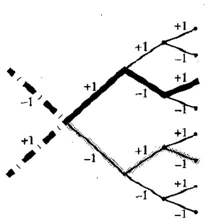

Fig. 3: Modified FDTS detection with MTR coding

With this modification in FDTS/DF detection, the squared minimum Euclidean distance between any two diverging paths, denoted by $\beta^2_{\min}$, is given by $4\cdot(1+f_1^{\,2}+f_2^{\,2}+\cdots+f_\tau^{\,2})$ for τ greater than or equal to 2, where f_k represents the equalized dibit response (at the output of the forward equalizer). For example, the effective SNR gain of the τ=2 FDTS/DF over the decision feedback equalization (DFE) channel, assuming the same MTR code, is given by $10\cdot\log_{10}(1+f_1^{\,2}+f_2^{\,2})$ dB.

The distance gain with MTR coding is also significant for high order PRML systems such as E^2PR4. When the critical NRZ error pattern is $\pm\{2\ -2\ 2\}$, the minimum distance for the E^2PR4 response {1 2 0 -2 -1} is $6\sqrt{2}$. With MTR coding, the worst case error pattern becomes a single bit error pattern of $\pm\{2\}$, and the corresponding channel output distance is simply the square root of the energy in the equalized dibit response, or $10\sqrt{2}$. This increase in the minimum distance is equivalent to an SNR gain of 2.218 dB. When the code rate penalty is small, the overall coding gain is significant.

IV. BER SIMULATION RESULTS

To verify the coding gain, FDTS/DF detection was simulated with the rate 4/5 and rate 16/19 MTR codes as well as with a rate 8/9 (0,k) code. The BERs were first obtained as a function of readback SNR for different tree depths. The BER of the PR4ML detector was also simulated for comparison. The Lorentzian transition response was assumed, and the user density, defined as PW50 over the user bit interval, is fixed at 2.5 for all codes. The SNR value required to achieve an error rate of 10^{-5} was then recorded for each depth/code combination.

The results are summarized in Fig. 4, where the effective SNR improvement of each system over PR4ML is shown. The performance advantage of MTR codes is clear. With the rate 16/19 MTR code, for example, the depth 1 FDTS/DF performs as well as the depth 5 FDTS/DF used with the conventional (0,k) code, yielding a 2.5 dB gain over the PR4ML. When the 4/5 MTR code is used, FDTS/DF with a tree depth of 2 outperforms the depth 5 FDTS/DF with the 8/9 (0,k) code. For a given tree depth, the rate 16/19 MTR code yields a 1.5 - 2 dB coding gain over the conventional 8/9 (0,k) code.

Also shown are the SNR performances of PRML systems with and without MTR coding. The coding gain is obvious with E^2PRML and E^3PRML, in which the minimum distance is improved with the MTR code. However, with EPR4ML the performance advantage of the MTR code is small since the MTR code does not improve the minimum distance in the EPR4 system. This is because the minimum distance error pattern in an EPR4 system is of the form $\pm\{2\}$, which is not affected by the MTR constraint. The MTR code does, however, eliminate non-minimum distance error patterns of the form $\pm\{...2\ -2\ 2...\}$, resulting in a small performance improvement over the (0,k) coded EPR4 system when the code rate is sufficiently high as with the 16/19 code.

Comparisons also can be made between the PRML systems and FDTS/DF systems. For example, the depth 2 FDTS/DF with the rate 4/5 MTR code improves more than 1 dB over EPR4ML with the rate 8/9 (0,k) code. At this density and with a Lorentzian transition response, EPR4ML has a 1.5 dB advantage over PR4ML. Of the PR targets, the EPR4 appears to provide a best fit to the natural channel as indicated by the superior performance of EPR4ML over even higher order PRML systems. Large enough FIR filters are used for equalization for both PRML and FDTS/DF systems so that the performances are not degraded by imperfect equalization.

In Fig. 5, similar plots are presented for a modeled MR head response. The trends are similar to the Lorentzian case, except that within the PRML family the performance improves as the order of the PR polynomial increases. Also, the MTR coding gain is larger than in the case of the Lorentzian response for all detectors. The depth 2 FDTS/DF channel with the rate 4/5 MTR code provides a 2.5 dB SNR gain over the EPR4ML channel with the rate 8/9 (0,*k*) code. With the particular MR head response used here, EPR4ML already has a 4 dB advantage over PR4ML at this linear density.

Since the MTR code eliminates data patterns with crowded transitions, the overall transition noise, as measured per unit length of track, is expected to be reduced. Fig. 6 shows the simulation results similar to those presented in Fig. 5, except random transition position jitter and transition width variations are included in the read waveform construction process [11]. The rms values of both transition noise parameters are set at 4.4 % of the user bit interval. The SNR reflects only the additive noise component. As is evident from the figure, the coding gain of the MTR code over the (0,*k*) code is much larger in the presence of transition noise. For example, with τ=2 FDTS/DF detection, the SNR difference is 6 dB between the rate 4/5 MTR code and the rate 8/9 (0,*k*) code which allows long runs of consecutive transitions.

Although the results are not shown here, we have also observed that the MTR code tends to reduce the relative frequencies of long error events in DFE and FDTS/DF systems.

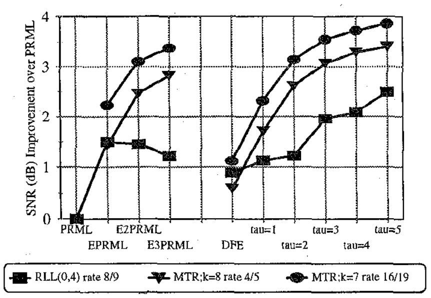

Fig. 4: Summary of PRML and FDTS/DF performances with and without MTR codes (Lorentzian response and additive noise).

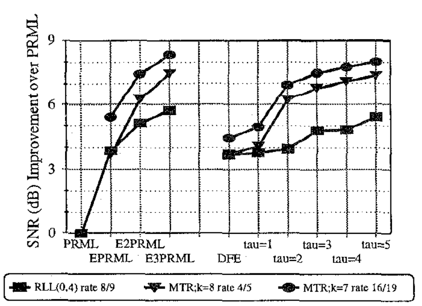

Fig. 5: Summary of PRML and FDTS/DF performances with and without MTR codes (MR head response and additive noise).

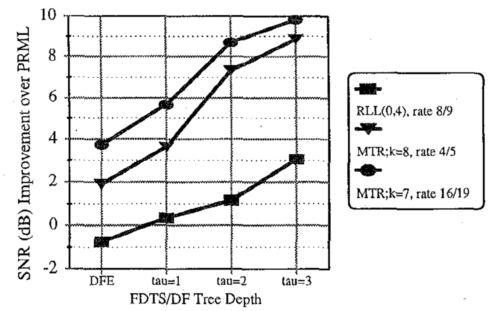

Fig. 6: Summary of FDTS/DF performances with and without MTR codes (MR head response and mixed noise).

V. CONCLUSION

A simple coding scheme is presented which improves the performance of FDTS/DF and high order PRML systems operating at relatively high linear densities. The code eliminates three or more consecutive transitions while allowing the *k*-constraint for timing purposes. The code can be implemented as simple block codes with reasonable rates such as 4/5, 8/10 and 16/19. BER simulations on FDTS/DF and PRML systems confirm large coding gains over the conventional (0,*k*) code.

REFERENCES

[1] B. Brickner and J. Moon, "Coding for increased distance with a d=0 FDTS/DF detector," Seagate Internal Report, May 1995; Also see, J. Moon and B. Brickner, "Coded FDTS/DF," presented at the Annual Meeting of the National Storage Industry Consortium, Monterey, CA, June 1995.

[2] J. Moon and B. Brickner, "MTR codes for Data Storage Systems," Invention Disclosure No. 96025, University of Minnesota, September 1995.

[3] B. Brickner and J. Moon, "A signal space representation of FDTS for use with a d=0 code," Globecom'95, Singapore, November 1995.

[4] K. A. S. Immink, "Coding techniques for the noisy magnetic recording channel," *IEEE Trans. Commun.*, vol. 37, no. 6, May 1989.

[5] J. Moon and J.-G. Zhu, "Nonlinearities in thin-film media and their impact on data recovery," *IEEE Transactions on Magnetics*, vol. 29, No. 1, Jan. 1993.

[6] E. Soljanin, "On-track and off-track distance properties of class4 partial response channels," SPIE Conference, Philadelphia, PA, Oct. 1995.

[7] R. Karabed and P. H. Siegel, "Coding for high order partial response channels," SPIE Conference, Philadelphia, PA, Oct. 1995.

[8] P. H. Siegel, "Recording codes for digital magnetic storage," *IEEE Transactions on Magnetics*, vol. MAG-21, no. 5, pp. 1344 - 1349, Sept. 1985.

[9] J. Moon and L. R. Carley, "Performance Comparison of Detection Methods in Magnetic Recording," *IEEE Transactions on Magnetics*, vol. 26, no. 6, Nov. 1990.

[10] B. Brickner and J. Moon, "A high dimensional signal space implementation of FDTS/DF," presented at Intermag '96, Seattle, Washington, April 1996.

[11] J. Moon, "Discrete-time modeling of transition-noise-dominant channels and study of detection performance," *IEEE Transactions on Magnetics*, vol. 27, no. 6, Nov. 1991.