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# The Geometry of Automotive Rearview Mirrors - Why Blind Zones Exist and Strategies to Overcome Them

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## ABSTRACT

Equations are derived which describe and relate the magnification, viewing angle and reflected illuminance of convex mirrors as used in automotive applications. The derived equations are compared to those for plane mirrors. Using these equations, the viewing angles of automotive rearview mirrors are calculated and depicted. The blind zones are defined in terms of the viewing angles, obstructions to vision, perceptibility limitations, and the lateral separation of vehicles. Various strategies for overcoming the blind zones are discussed.

## INTRODUCTION

The blind zones produced by automotive rearview mirrors have long been of concern. A variety of ways of coping with them are in use, and new ways are being proposed. Blind zones are an important factor in accidents caused by lane changing maneuvers. The National Highway Traffic Safety Administration (NHTSA) Crash Avoidance Research Program has targeted Lane Change/Merge (LCM) crashes as one of five categories of crashes potentially suitable for high technology Intelligent Vehicle Highway System (IVHS) crash avoidance countermeasures (Knipling, 1993). Wang and Knipling (1994) estimate that LCM crashes account for 4.0% of passenger car crashes, 225 fatalities and 630,000 crashes annually. About 50% of these occur on urban divided highways, and about 75% occur during daylight hours. Involvement by direction of lane change is about equally split between right to left and left to right changes. An analysis of LCM crashes by Najm et. al., 1994 shows that in 61.2% of crashes, the driver did not see the other vehicle. In 29.9% of crashes, the driver misjudged the position and /or speed of the other vehicle.

Mirror blind zones are not responsible for all of the LCM type crashes. However, they are extremely important in that they are not well understood by the average driver, and yet they are an integral part of the data acquisition system used by drivers in LCM maneuvers. Understanding the blind zones

is important, and key elements in understanding them are the viewing angles of the mirrors and where the views are directed.

Equations will be derived which quantify the viewing angles in terms of the relevant parameters. Much of the derivations are by way of review, but some new relationships are shown. Graphical methods can be used to show viewing angles, but analytical methods bring out insights not obtainable graphically. Hence the analytical approach.

The derived equations are next used to calculate the viewing angles of the mirrors on a vehicle using their dimensions and their positions relative to the driver. Then the blind zones are defined and depicted. Several factors in addition to the viewing angles determine the extent of the blind zones, including obstructions to vision due to the vehicle, the driver's ability to perceive objects in both the mirror and his or her peripheral vision, and the lateral separation of the vehicles on the roadway.

Once the blind zones have been established, various strategies which have been developed to overcome the blind zones will be reviewed.

## GEOMETRY OF REARVIEW MIRRORS

On US passenger cars, the inside mirror and left outside mirror are plane, and the right outside mirror is convex. Europe and Japan allow the use of convex mirrors on the driver's side. Since a plane mirror is a special case of a convex mirror with an infinite radius, the convex mirror equations will be derived first.

The spherical convex mirror presents an image at a magnification less than unity. Most introductory physics books give the equations relating the mirror radius, the object distance, the image distance and the magnification. For the spherical convex mirror of Figure 1, it is easily shown that;

$$\frac{1}{-p} + \frac{1}{-q} = \frac{2}{r} = \frac{1}{f} \quad \text{EQ(1)}$$

and,

$$m = \frac{q}{p}, \quad \text{EQ(2)}$$

where,  $p$  = distance of the object from the mirror  
 $q$  = distance of the image from the mirror  
 $r$  = radius of the mirror  
 $f$  = focal length of the mirror  
 $m$  = magnification.

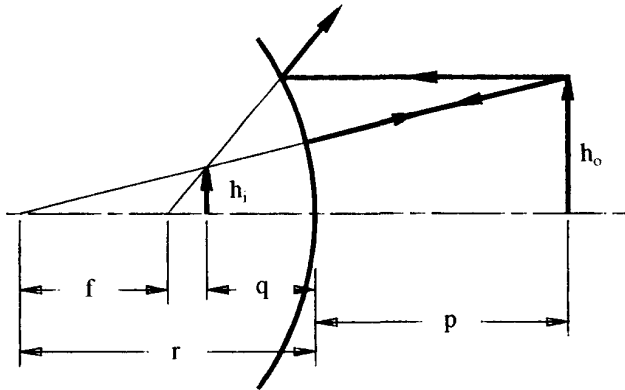


Figure 1

In Figure 1, distances to the right of the mirror are positive and distances to the left are negative. The image distance and the radius will be negative for the convex mirror. The image of the convex mirror is erect and virtual, i.e., an object in the mirror appears standing right side up, and the image cannot be focused on a screen. Concave mirrors have inverted real images which may be focused on a screen.

Since we want to know what an image in the mirror looks like to the driver, let's begin by calculating the image distance and height of an object from EQ(1). The image distance is;

$$q = \frac{rp}{2p-r} \quad \text{EQ(3)}$$

The image height is;

$$h_i = mh_o = \frac{q}{p}h_o, \quad \text{EQ(4)}$$

where  $h_o$  is the height of the object. Substituting EQ(3) into EQ(4), shows that;

$$h_i = \frac{r}{2p-r} h_o. \quad \text{EQ(5)}$$

Note that  $r$  has negative values for the convex mirror and that  $h_i$  does not go to infinity when  $p = r/2$ . The magnification is:

$$m = \frac{h_i}{h_o} = \frac{r}{2p-r}. \quad \text{EQ(6)}$$

The magnification is seen to be a function of the object distance. As a numerical example, choose  $r = -0.5$  ft. and  $p = 5.0$  ft. Then  $m = -0.5$ . How does this compare to a plane mirror? For a plane mirror  $r = \infty$ , and  $m$  is always -1.

Now we know that the convex mirror produces a virtual image smaller than that of a plane mirror and that the image size is a function of the object distance from the mirror. The eye forms an image on its retina of the mirror's virtual image. We could calculate the overall magnification from the object height to the retinal image height if we assumed a "standard" eye with an associated "standard" focal length lens and lens to retina distance. However, as drivers or mirror engineers, we really only want to know how the eye sees the convex mirror image compared to a plane mirror image, i.e., how much smaller is the image from a convex mirror than the image from a plane mirror. To do this, we only need to compare the angles subtended from the eye to the mirror virtual images. This is because the retinal image height is proportional to the subtended angle.

Figure 2 shows the subtended angle,  $\phi$ , for a convex mirror. For simplicity, the eye and the object being viewed are on the axis of the mirror. The eye is shown at a distance  $s$  from the mirror. The construction lines show the image of the object and the angle subtended at the eye by the image.

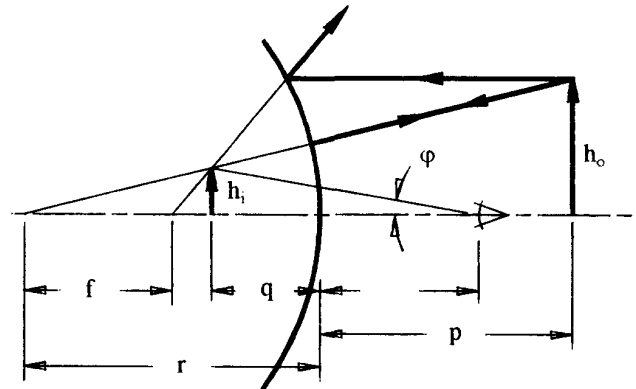


Figure 2

From Figure 2 it is seen that the subtended angle  $\phi$  is;

$$\phi = \tan^{-1} \frac{h_i}{s-q} = \tan^{-1} \frac{rh_o}{(s-q)(2p-r)}$$

$$\phi = \tan^{-1} \frac{h_o}{\frac{2sp}{r} - (s+p)}. \quad \text{EQ(7)}$$

EQ(7) is the subtended angle for a convex mirror, and letting  $r$  go to infinity in EQ(7) gives the subtended angle for a corresponding plane mirror. The relative magnification of the convex mirror compared to the plane mirror will be denoted by  $m_R$ , and it is:

$$m_R = \frac{\tan^{-1} \frac{h_o}{\frac{2sp}{r} - (s+p)}}{\tan^{-1} \frac{h_o}{-(s+p)}} \quad \text{EQ(8)}$$

While precise, EQ(8) is difficult to interpret at a glance. This can be helped by recalling that ;

$$\tan^{-1} \chi = \chi - \frac{\chi^3}{3} + \frac{\chi^5}{5} - \dots \quad \text{EQ(9)}$$

For an automotive mirror,  $r \cong -5$  ft,  $s \cong 4$  ft, and  $h_o$  is at most  $1/3$  of  $p$ . In this case, the higher order terms are extremely small and they may be ignored. Then;

$$m_R \cong \frac{1}{1 - \frac{2sp}{r(s+p)}} \quad \text{EQ(10)}$$

Finally, we have a simple understandable equation that compares what we would see in a convex mirror with what we would see in a like plane mirror.

Figure 3 is a graph of  $m_R$  vs  $p$  for  $r = -5$  ft and  $s = 4$  ft. It is seen that  $m_R$  goes to unity at  $p = 0$  and to .38 at  $p = \infty$ . As a car approaches, it appears to increase in size at a faster rate than would a car in a plane mirror.

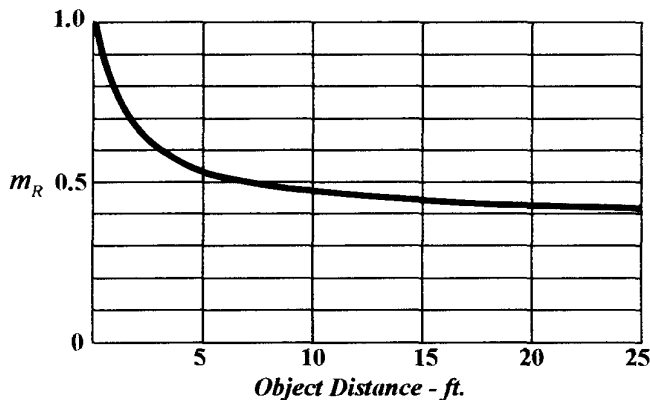


Figure 3

Next, an expression for the viewing angle of a convex mirror will be derived. Figure 4 shows a convex mirror of radius,  $r$ , and width,  $w$ , in a horizontal plane to depict the horizontal viewing angle. An observer is shown at a distance  $s$  from the mirror. The incoming ray shown defines the widest angle that can be seen for the mirror width and eye position depicted. The total viewing angle of the mirror will

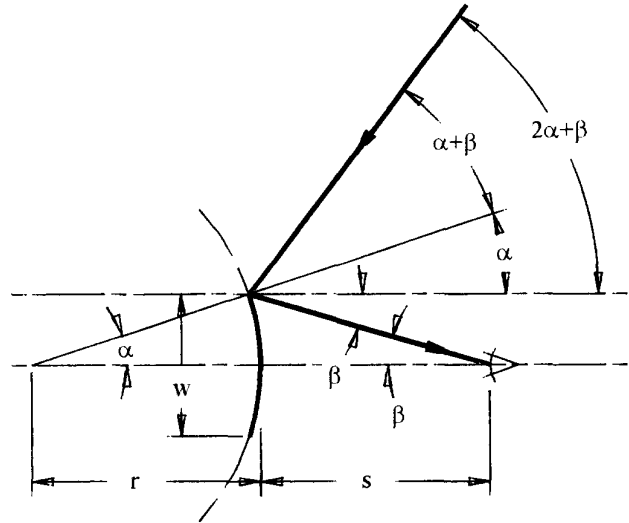


Figure 4

be twice the angle that the incoming ray makes with the axis line. This angle is obviously:

$$\theta = 2(2\alpha + \beta) \quad \text{EQ(11)}$$

Noting that,

$$\alpha = \tan^{-1} \frac{w}{2r} \quad \text{EQ(12)}$$

and,

$$\beta = \tan^{-1} \frac{w}{2s} \quad \text{EQ(13)}$$

then,

$$\theta = 2 \left[ \tan^{-1} \frac{w}{2r} + \tan^{-1} \frac{w}{2s} \right] \quad \text{EQ(14)}$$

It is of interest to compare this angle with the viewing angle of a plane mirror of the same width at the same position. The ratio of the convex mirror viewing angle to the plane mirror viewing angle will be denoted by  $R$ , and it is;

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