Estimating Position and Velocity of Mobiles in a Cellular Radio Network

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Abstract— Determining the position and velocity of mobiles is an important issue for hierarchical cellular networks since the efficient allocation of mobiles to large or microcells depends on its present velocity. In this paper, we suggest a method of tracing a mobile by evaluating subsequent signal-strength measurements to different base stations. The required data are available in the global system for mobile (GSM) system. The basic idea resembles multidimensional scaling (MDS), a well-recognized method in statistical data analysis. Furthermore, the raw data are smoothed by a linear regression setup that simultaneously yields an elegant, smoothed estimator of the mobile's speed. The method is extensively tested for data generated by the simulation tool GOOSE.

I. INTRODUCTION

RAPIDLY growing load is expected for cellular radio networks in the near future. Since the number of channels, either physical, time division multiple access (TDMA), or code division multiple access (CDMA), is a restricted resource for such systems, increasing demand can only be satisfied by diminishing the transmission power and coverage area of cells. This, however, increases the number of handoffs and the corresponding administration overhead. Hierarchical cell structures seem to be a good compromise between an efficient use of available channels while simultaneously keeping the number of handoffs small. Fast-moving mobiles are assigned to larger cells, and stationary or slow-moving stations are allocated to microcells, which are used in areas of high utilization. Of course, physical channels must be segregated in overlapping cell areas.

For an efficient channel assignment, information about the position and velocity of mobile stations is inevitable in hierarchical networks. External data, e.g., from the global positioning system (GPS), could be used to calculate and transmit the present position and velocity in subsequent time slots. This has two main disadvantages. First of all, the size and cost of mobile transmitters would be considerably increased because of the incorporated GPS receiver. Moreover, a clear view of the sky is necessary to receive usable GPS data of at least three satellites, which makes the system useless in buildings or roads surrounded by high buildings or mountains. Thus, a system is preferable that can be implemented within the existing GSM standard, independent of external information.

Two quantities are near at hand to obtain distance and speed information: signal strengths of different base stations measured at a mobile and corresponding propagation times. Both parameters are subject to strong fluctuations caused by short-term fading, shadowing, and reflections such that a sophisticated method is necessary to translate signal strength into distance information. Several procedures have been suggested in literature.

First attempts to monitor the position of vehicles arose from the need for knowing the disposition and status of vehicles in transport systems (see [16] and [17] for an overview). Reference [3] describes a system where the signal strength of a mobile's transmitter is measured on a statistical basis by a set of base stations. From *a priori* information of the corresponding contours, the most probable location of the mobile is determined. Concerning the basic idea, this approach is related to what we will develop in our paper. However, in [3] no feasible search procedure for realistic scenarios is offered.

Using elementary geometric considerations and a least squares estimate to smooth measurement errors, [19] develops a trilateration method based on radio-frequency (RF) travel time measurements between the vehicle and fixed sensors located at the edges of a square. This method was further pursued by [4] for channel allocation in cellular networks. Refined trilateration techniques for road environments, using time delays between reception of a mobile's transmission at different nodes as input data, are investigated by [21].

Signal-strength measurements are used in [6] to assign a mobile to a certain base station coverage zone. A channel allocation algorithm is introduced that uses this information as a basic ingredient. In [2], the area of interest is divided into small sub areas that are (not uniquely) characterized by a list of discrete signal power values from different base stations. However, a nonsatisfying behavior of the system is reported if complicated shadowing environments are considered.

Recently, in [9] and [10], adaptive schemes based on hidden Markov models, neural networks, and pattern-recognition methods have been employed to estimate the position of mobiles.

Some work has also been devoted to the estimation of velocity only. The elapsed time until a cell handoff in a picocell occurs yields a rough estimate of a mobile's speed and forms the basis for a handover algorithm in [20]. Reference [8] uses the number of level crossings of the average signal level to estimate the velocity. Reference [12] develops a method

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for velocity estimation if diversity reception is available. The authors determine the speed of a mobile via the estimated expected diversity branch switching rate between two diversity branches using the Doppler effect.

In this paper, we use a method called multidimensional scaling (MDS) in data analysis literature (see [1], [14], and [18]). According to a least squares criterion, a point in the area of interest is determined in such a way that the measured signal strength of certain base stations is best fitted to the known average signal strength at this point. The necessary data are available in the GSM system, where each 0.48 s the downlink signal levels of six neighboring base stations are transmitted on a discrete scale from 0–63. A fast algorithm and a corresponding smoothing procedure are developed that allow the online estimation of the position and velocity of mobiles with high accuracy. The performance of our method is tested by data for different complicated scenarios. These are generated by the powerful, close-to-reality simulation tool GOOSE, whose basic principles are briefly outlined in Section II.

II. Estimating Positions via Signal Strength Measurements

To clarify the concept, we first introduce our method under simplifying assumptions. Let $\mathbf{z}_i \in \mathbb{R}^2$ denote the position of base station i and $d_i(\mathbf{x}) = \|\mathbf{x} - \mathbf{z}_i\|$ the Euclidean distance of $\mathbf{x} \in \mathbb{R}^2$ from $\mathbf{z}_i$. If the average signal strength follows a propagation law of the type $cd^{-\alpha}$, d denoting the distance from the transmitter, c a constant, and $\alpha \geq 2$ the attenuation exponent, then the average signal strength $s_i(\mathbf{x})$ of base station i measured at position $\mathbf{x}$ is obtained as

$$s_i(\mathbf{x}) = c[d_i(\mathbf{x})]^{-\alpha}. \tag{1}$$

Let $\gamma_i(t)$ denote the measured signal strength of base station i, $i = 1, \cdots, n$, at a certain position. Obviously, $\gamma_i(t)$ is subject to random fluctuations due to short-term Rayleigh and Rice fading. The transformed values $\gamma_i^\alpha(t)/c = \delta_i(t)$ correspond to the distance from base station i, such that the solution $\hat{\mathbf{x}}(t)$ of

$$\text{minimize} \quad f(\mathbf{x}) = \sum_{i=1}^n [d_i(\mathbf{x}) - \delta_i(t)]^2 \quad \text{over} \quad \mathbf{x} \in \mathbb{R}^2 \tag{2}$$

is a least squares estimator of the actual position at time t.

This is actually a typical question of MDS. Given certain pairwise pseudo distances or dissimilarities δ_{ij} between n objects, MDS aims at determining a representation of the objects by n points in a k-dimensional Euclidean space such that the interpoint distances fit the given dissimilarities optimally. Here we have the special case that the positions of the base stations are *a priori* fixed, and only the position of the mobile has to be fitted to the transformed signal data.

A local minimum of (2) can be calculated numerically by a Newton-type iteration

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \mathbf{H}_f^{-1}(\mathbf{x}_k) \nabla f(\mathbf{x}_k) \qquad k \in \mathbb{N}_0. \tag{3}$$

The gradient $\nabla f(\mathbf{x})$ at differentiable points $\mathbf{x}$ is given by

$$\nabla f(\mathbf{x}) = 2 \sum^n \frac{d_i(\mathbf{x}) - \delta_i(t)}{} \tag{4}$$

and with $\mathbf{x} = (x_1, x_2)'$ and $\mathbf{z}_i = (u_i, v_i)'$, the Hessian is determined as

$$\mathbf{H}_f(\mathbf{x}) = \begin{bmatrix} f_{11}(\mathbf{x}) & f_{12}(\mathbf{x}) \\ f_{12}(\mathbf{x}) & f_{22}(\mathbf{x}) \end{bmatrix} \tag{5}$$

where

$$f_{11}(\mathbf{x}) = 2 \sum_{i=1}^n \frac{\delta_i(t)(x_1 - u_i)^2}{d_i^3(\mathbf{x})} + \frac{d_i(\mathbf{x}) - \delta_i(t)}{d_i(\mathbf{x})}$$

$$f_{22}(\mathbf{x}) = 2 \sum_{i=1}^n \frac{\delta_i(t)(x_2 - v_i)^2}{d_i^3(\mathbf{x})} + \frac{d_i(\mathbf{x}) - \delta_i(t)}{d_i(\mathbf{x})}$$

$$f_{12}(\mathbf{x}) = 2 \sum_{i=1}^n \frac{\delta_i(t)}{d_i^3(\mathbf{x})}(x_1 - u_i)(x_2 - v_i).$$

Iteration (3) solves $\nabla f(\mathbf{x}) = 0$. A sufficient condition for a local minimum at a stationary point $\mathbf{x}$ is $\mathbf{H}_f(\mathbf{x})$ positive definite, which is checked in the following numerical examples.

It is well known that algorithm (3) is prone to get stuck at local minima and does not necessarily find the global minimum. However, a reliable estimation $\hat{\mathbf{x}}(t_i)$ of a mobile's position at time t_i is a reasonable starting point for finding the next solution $\hat{\mathbf{x}}(t_{i+1})$ by (3).

The above described procedure has been applied to data generated by GOOSE, a tool to simulate radio wave propagation for cellular radio networks in realistic environments. The basic input data fed to Goose are: 1) topographical and morphological data; 2) mobility patterns; and 3) the cellular network configuration. Radio wave propagation is simulated on the basis of this input and follows the Okumura–Hata model [5], [15], taking account of reflections, shadowing, and fading. GOOSE is developed at ComNets at the Aachen University of Technology.

The area of interest is a square of 10×10 km that contains six base stations located according to Fig. 1. A plane landscape is assumed such that a simple propagation law with circle-shaped contour lines of locations of identical signal strength applies. A mobile is moving on a straight line from the left to the right margin of the area with 100 km/h, as is depicted by the solid line in Fig. 1. This enables measurements during a time period of 6 min, one each 0.48 s, in total $m = 748$ blocks of $n = 6$ signals strengths to each of the base stations.

GSM mobiles code measurements of signal strength on a discrete scale from 0–63 (7 b). This relatively rough scale makes the retransformation to distances a bit more complicated and yields extra errors. For instance, the received coded signal strength from base station 2 over 360 s is depicted in Fig. 2. However, the test results show that our algorithm copes easily with these extra inaccuracies.

Applying the Newton iteration (3) to each of the 748 data sets, $[\delta_1(t_i), \cdots, \delta_6(t_i)]$, $i = 1, \cdots, 748$, results in a sequence of successing positions that are plotted in Fig. 1. It can be clearly seen that estimation is more accurate in the middle where the mobile is surrounded by base stations. The starting point for the first localization by (3) is randomly chosen according to a uniform distribution on the relevant area. In each subsequent optimization, the preceding estimation is used as a starting point in (3). Iteration scheme (3) is stopped
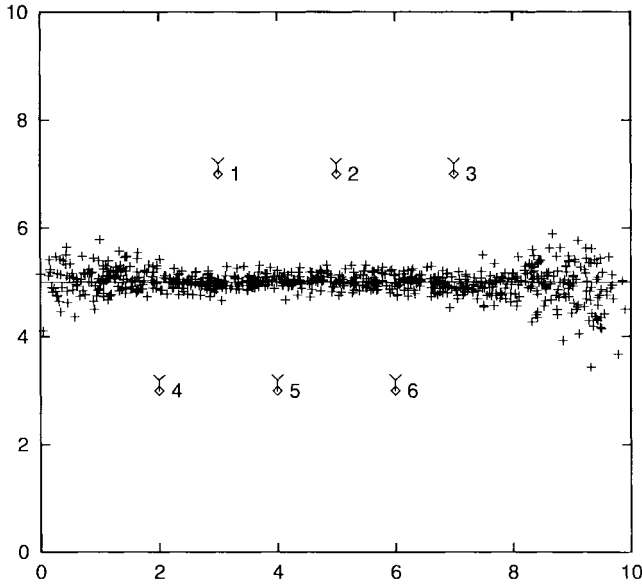
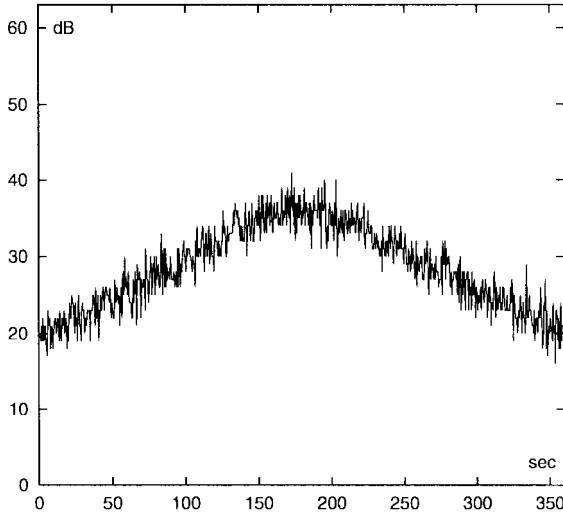Fig. 1.   Scenario, true track, and estimated positions (nonsmoothed).

Fig. 2.   Received signal strength of base station 2.

III. SMOOTHING ESTIMATED TRACKS AND VELOCITY

Due to random fading of the signal, the estimated track of the mobile based on $\hat{\mathbf{x}}(t_i)$, $i = 1, \cdots, m$ is rather zigzagged. This can be clearly seen from Fig. 1. The real track of a mobile can be approximated more closely by smoothing the data. We suggest a linear regression setup that has proved very efficient in our numerical experiences. Let

$$\hat{\mathbf{x}}(t_i) = [\hat{x}_1(t_i), \, \hat{x}_2(t_i)]', \qquad i = 1, \cdots, m$$

denote the estimated locations at subsequent time points t_i and $i \in \mathbb{N}$. For the GSM system, we have $t_i = 0.48i$ s when normalizing $t_0 = 0$. It is assumed that the motion of the mobile can be (at least locally) approximated by a straight line. Since mobiles must be localized in real time, a smoothing algorithm at time t_i may only use past values at times t_j for $j \leq i$. We restrict our attention to k preceding estimated positions and

If the motion of the mobile is linear with constant speed vector $\mathbf{a}$ and velocity $\|\mathbf{a}\|$, then the true position at time t_i is given by

$$\mathbf{x}(t_i) = t_i \mathbf{a} + \mathbf{b}$$

where $\mathbf{b}$ is the position at time $t_0 = 0$. The parameters $\mathbf{a}$ and $\mathbf{b}$ in this linear regression setup are estimated from the observed values $\hat{\mathbf{x}}(t_1), \cdots, \hat{\mathbf{x}}(t_k)$ by the solutions $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ of the least squares criterion

$$\text{minimize} \sum_{j=1}^{k} \|\hat{\mathbf{x}}(t_j) - (t_j \mathbf{a} + \mathbf{b})\|^2 \quad \text{over } \mathbf{a}, \mathbf{b} \in \mathbb{R}^2. \quad (6)$$

Minimization problem (6) decomposes into

$$\min_{a_1, b_1 \in \mathbb{R}} \sum_{j=1}^{k} [\hat{x}_1(t_j) - (t_j a_1 + b_1)]^2$$

$$+ \min_{a_2, b_2 \in \mathbb{R}} \sum_{j=1}^{k} [\hat{x}_2(t_j) - (t_j a_2 + b_2)]^2.$$

From standard linear regression in statistics, the solution of either sum is well known to be

$$\hat{a}_\ell = \frac{\displaystyle\sum_{j=1}^{k} (t_j - \bar{t})[\hat{x}_\ell(t_j) - \bar{x}_\ell]}{\displaystyle\sum_{j=1}^{k} (t_j - \bar{t})^2}$$

$$\hat{b}_\ell = \bar{x}_\ell - \hat{a}_\ell \bar{t}, \quad \ell = 1, \, 2 \quad (7)$$

with $\bar{t} = (1/k) \sum_{j=1}^{k} t_j$, $\bar{x}_\ell = (1/k) \sum_{j=1}^{k} \hat{x}_\ell(t_j)$, and $\ell = 1, \, 2$. Hence, a solution of (6) is given by (7) through

$$\hat{\mathbf{a}} = (\hat{a}_1, \, \hat{a}_2)'$$
$$\hat{\mathbf{b}} = (\hat{b}_1, \, \hat{b}_2)'. \quad (8)$$

Now, let $\hat{\mathbf{a}}(t_i)$, $\hat{\mathbf{b}}(t_i)$ denote the solution (8) based on the last k measurements before and including time t_i, i.e., $t_{i-k+1}, \cdots, t_i$ and $\hat{\mathbf{x}}(t_{i-k+1}), \cdots, \hat{\mathbf{x}}(t_i)$. The actual position at time t_i is then estimated by the value of the regression line at $t = t_i$, i.e.,

$$\hat{\hat{\mathbf{x}}}(t_i) = t_i \hat{\mathbf{a}}(t_i) + \hat{\mathbf{b}}(t_i). \quad (9)$$

Differentiating $\hat{\hat{\mathbf{x}}}(t)$ at $t = t_i$ simultaneously yields a smoothed estimator of the instantaneous speed vector at time t_i, namely

$$\hat{\mathbf{s}}(t_i) = \hat{\mathbf{a}}(t_i), \qquad i = 1, \cdots, m. \quad (10)$$

The corresponding velocity is obtained as

$$\hat{v}(t_i) = \|\hat{\mathbf{a}}(t_i)\|$$
$$= [\hat{a}_1^2(t_i) + \hat{a}_2^2(t_i)]^{1/2}. \quad (11)$$

k is usually small with values between 5–20. To calculate the first $k - 1$ regression coefficients

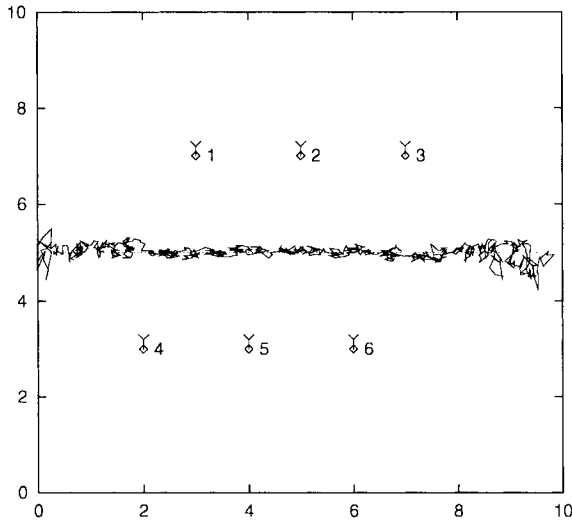Fig. 3. Regression-smoothed estimated track.

Fig. 4. Isoclines of signal strength of base station 2 with shadowing by hills (indicated by circles).

at time $t_{-k+2}, \cdots, t_0$ are missing. The necessary values are set equal to the first MDS-estimated position, i.e.,

$$
\begin{aligned}
\hat{\mathbf{x}}(t_{-k+2}) &= \cdots \\
&= \hat{\mathbf{x}}(t_0) \\
&= \hat{\mathbf{x}}(t_1).
\end{aligned}
$$

The linear smoothing procedure with duplicated initial estimators has been applied to the above example. The corresponding estimated track is represented in Fig. 3. It shows a very satisfying behavior. Estimated positions could be even improved by additional information on the topography. If the position of roads is available from an internal map of the relevant area, better estimators are obtained by projecting $\hat{x}(t_i)$ onto the closest road. Such additional information on the environment has been used for pattern-recognition techniques in [9].

IV. SHADOWING AND REFLECTIONS

The above procedure presupposes an exact knowledge of the average signal strength following a simple propagation law, e.g., (1). In reality, however, this is quite rarely the case. Due to shadowing and reflections, the contour lines of points of equal average signal strength are strongly deformed in an environment with hills and large buildings. The isoclines are no longer a simple function of the distances alone, as can be clearly seen from Fig. 4. The scenery considered here is the same as above, i.e., a square of 10×10 km with six base stations, but now two hills north and north-east of base stations 5 and 6, respectively, are inserted. The contour lines surrounding base station 2 are heavily influenced by this topography.

However, there exist computer programs to predict the average signal power quite accurately and also in complicated environments [7], [11]. Even a three-dimensional (3-D) model is available in [13]. These tools may be used to determine the average signal strength at any position in certain scenarios. Moreover, such data can also be obtained from real
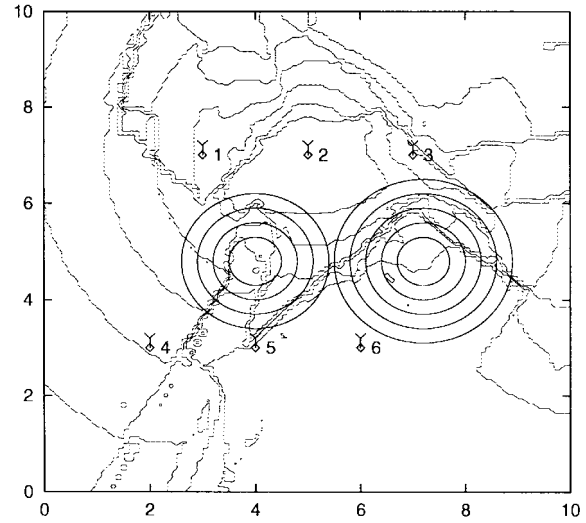
In the following, we assume that the average signal power $s_i(\mathbf{x})$ is known for each base station $i = 1, \cdots, n$ and for each location $\mathbf{x} \in A \subseteq \mathbb{R}^2$ of the relevant area. Let $\gamma_i(t)$, $i = 1, \cdots, n$ denote the measured signal strength of base station i at a certain position. To estimate this position, a modified problem of type (2) arises, namely

$$
\text{minimize} \quad g(\mathbf{x}) = \sum_{i=1}^{n} [s_i(\mathbf{x}) - \gamma_i(t)]^2 \qquad \text{over } \mathbf{x} \in A. \quad (12)
$$

A corresponding solution $\hat{\mathbf{x}}(t) = [\hat{x}_1(t), \hat{x}_2(t)]'$ estimates the position of a mobile at time t.

To solve (12), complete knowledge of the irregular average power surfaces $s_i(\mathbf{x})$: $A \to \mathbb{R}^+$ is necessary. The average signal power may be conveniently represented by a spline $s_i(\mathbf{x})$ with support points $(\mathbf{y}, s_{\mathbf{y}})$, where $\mathbf{y}$ is chosen on a dense grid $\mathcal{G} \subset \mathcal{A}$ in the relevant area and $s_{\mathbf{y}}$ is obtained from *a priori* measurements or simulations of the average signal power as described above. A Newton iteration of type (3) yields a (local) minimum of (12), where the gradient and Hessian are determined according to the spline representation $s_i(\mathbf{x})$. $g(\mathbf{x})$ is continuous and differentiable at interior points of the covering triangles with gradient

$$
\nabla g(\mathbf{x}) = 2 \sum_{i=1}^{n} [s_i(\mathbf{x}) - \gamma_i(t)] \nabla s_i(\mathbf{x})
$$

and Hessian

$$
\mathbf{H}_g(\mathbf{x}) = 2 \sum_{i=1}^{n} \{ \nabla s_i(\mathbf{x})[\nabla s_i(\mathbf{x})]' + [s_i(\mathbf{x}) - \gamma_i(t)]\mathbf{H}_{s_i}(\mathbf{x}) \}.
$$

Substituting $s_i(\mathbf{x})$ by $d_i(\mathbf{x})$ and $\gamma_i(t)$ by $\delta_i(t)$ yields (4) and (5) as a special case.

This type of fitting the position via a spline function worked well in numerical examples, but it is relatively calculation insensitive and time consuming. For practical purposes, it is sufficient to solve (12) over $\mathbf{x}$ on a grid $\mathcal{L} \subset A$ with grid constant 25 m, say, in both coordinates. Then (12) is a
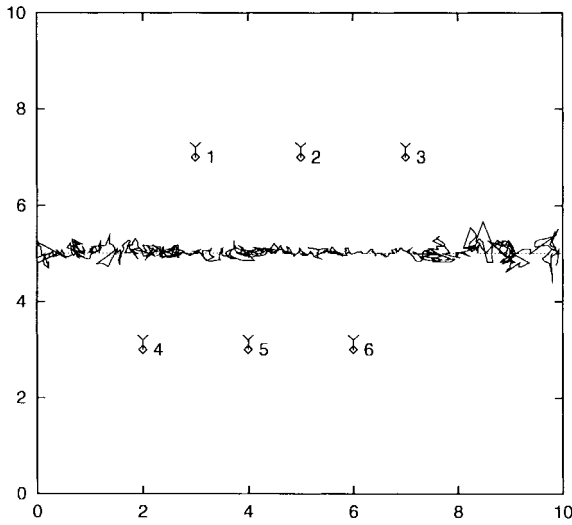
Fig. 5. Regression-smoothed estimated track with shadowing.

Fig. 6. Regression-smoothed estimated velocity with shadowing.

estimator $\hat{\mathbf{x}}(t_\ell)$, subsequent minima must be searched only over a local square of grid values in $\mathcal{L}$ surrounding $\hat{\mathbf{x}}(t_\ell)$, since the mobile's distance from $\hat{\mathbf{x}}(t_\ell)$ is bounded by the radius $v \cdot \Delta t$, v the maximum velocity, and Δt the time between subsequent measurements. The minimization can be carried out on-line by complete enumeration and yields a fast method to trace the mobile by estimates $\hat{\mathbf{x}}(t_i) \in \mathcal{L}$, $i = 1, \cdots, m$. Position and velocity are then determined by the smoothed regression estimators with (9) and (10) with $k = 15$. Fig. 5 shows the resulting track for the environment of Fig. 4. An average deviation of 95 m from the true track is observed that is quite satisfying for practical purposes. Fig. 6 shows the estimated velocity (in meters per second) over time (from 0 to 360 s), based on the regression-smoothed values (11). Although averaging over the last 15 measurements, the estimation error is still quite large. Large deviations from the constant speed of 27.8 m/s occur, demanding further smoothing.

V. TEST RESULTS FOR NONSTRAIGHT MOTION AND FURTHER SMOOTHING

The above described procedure, based on a linear regression setup, is adapted to at least locally straight motion. It is important to see how bended tracks influence its performance. For this purpose we consider a relevant area of 10×10 km with seven base stations, whose positions are depicted in Fig. 7. Moreover, the average signal-strength isoclines of base station 2 are included, corresponding to two hills located at coordinates $(0, 0)$ and $(4.8, 4.8)$. A mobile is moving from the left to the right margin with a constant speed of 100 km/h on a track containing a quarter right, two-quarter left, and again a
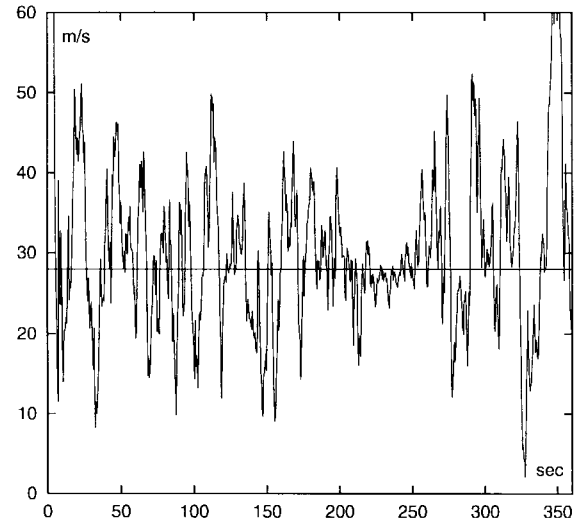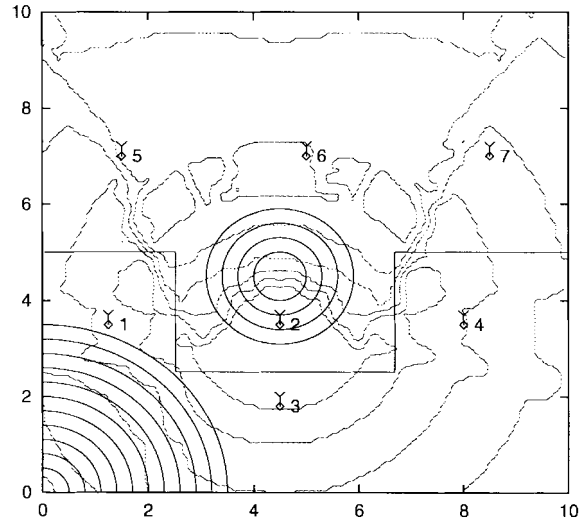

Fig. 7. Base stations, the mobile's track, and isoclines of base station 2.

quarter right turn. The mobile can be observed for 9 min in the relevant area. Its way is represented by a solid line in Fig. 7.

Successive positions $\hat{\mathbf{x}}(t_i)$, $i = 1, \cdots, 1120$ have been estimated by solving the discrete optimization problem described in Section IV. Based on these values, the regression-smoothed estimators $\hat{\mathbf{x}}(t_i)$ from (9) are calculated and successively connected by lines. The corresponding estimated track of the mobile is depicted in Fig. 8. The average deviation from the true track is 90 m. In spite of three bends and constant speed while turning, the estimators are quite accurate and reliable.

By now we have not used the fact that a mobile of maximum speed $v_{\max}$ cannot leave the circle of radius $r_{\max} = t \cdot v_{\max}$ surrounding its present position within time t. This information

$$\overline{\hat{\mathbf{x}}}(t_{n+1}) = \begin{cases} \hat{\mathbf{x}}(t_{n+1}), & r_{\max} \qquad \text{if } \|\mathbf{x}^*(t_n) - \hat{\mathbf{x}}(t_{n+1})\| \leq r_{\max} \end{cases} \qquad (13)$$

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