LEON W. COUCH II

Professor of Electrical Engineering
University of Florida, Gainesville

DIGITAL AND ANALOG COMMUNICATION SYSTEMS

Second Edition

Macmillan Publishing Company
New York

Collier Macmillan Publishers
London

Macmillan Publishing Company
866 Third Avenue, New York, New York 10022

Collier Macmillan Canada, Inc.

Library of Congress Cataloging in Publication Data

Couch, Leon W.
Digital and analog communication systems.

Bibliography: p.
Includes index.
1. Telecommunication systems. 2. Digital communications. I. Title.
TK5101.C69 1987 621.38'0413 86-8739
ISBN 0-02-325380-0

Printing: 1 2 3 4 5 6 7 8 Year: 7 8 9 0 1 2 3 4 5

ISBN 0-02-325380-0

To my wife,
Margaret Wheland Couch,
and
To our children,
Leon III, Jonathan, and Rebecca

PREFACE

This new edition provides the latest up-to-date treatment of digital and analog communication systems. It is written as a textbook for junior or senior engineering students and is appropriate also for an introductory graduate course or as a modern technical reference for practicing electrical engineers. It covers *practical aspects* of communications systems developed from a sound *theoretical basis*.

The Theoretical Basis

- Representation of digital signals
- Representation of analog signals
- Magnitude and phase spectra
- Fourier analysis
- Power spectral density
- Linear systems
- Nonlinear systems
- Modulation theory
- Random variables
- Probability density
- Random processes
- Calculation of (S/N)
- Calculation of BER
- Optimum systems

- Simulation of communication systems
- Intersymbol interference
- Block codes
- Convolutional codes

The Practical Applications

- PAM, PCM, DPCM, DM, PWM, and PPM baseband signaling
- OOK, BPSK, QPSK, MPSK, MSK, and QAM bandpass digital signaling
- AM, DSB-SC, SSB, VSB, PM, and FM bandpass analog signaling
- Time division multiplexing and the standards used
- Frequency division multiplexing and the standards used
- Common carrier systems
- Satellite communication systems
- Effective input-noise temperature and noise figure
- Link budget analysis
- Fiber optic systems
- Spread spectrum systems
- Television systems
- Technical standards for AM, FM, TV, and CATV
- Computer communication systems
- Protocals for computer communications
- Technical standards for computer communications
- Math tables
- Illustrative examples
- 383 homework problems with selected answers
- Extensive references

The practical aspects are exhibited by describing the circuitry that is used in communication systems and summarizing the technical standards that have been adopted for digital, analog, and computer communication systems. The theoretical aspects are presented by using a sound mathematical basis that is made clear by the use of a definition, theorem, proof format with worked examples.

This book is an outgrowth of my teaching of graduate as well as undergraduate communication courses at the University of Florida. I believe that the reader does not fully understand the technical material unless he or she has the opportunity to work problems. Consequently, over 380 problems have been included. Some of them are easy so that the beginning student will not become frustrated, and some are difficult enough to challenge the more advanced student. All the problems are designed to provoke thought about, and understanding of, communication systems.

The contents of this book may be divided into two main parts. Chapters 1 through 5 develop communication systems from a nonrandom signal viewpoint. This allows the reader to grasp some very important ideas without having to learn (or know) statistical concepts at the same time. In the second part of the book, Chapters 6 through 8 plus Appendix B, statistical concepts are developed and used. Statistical concepts are needed to analyze and design communication systems that are operating in the presence of noise. Some sections of Chapters 1 through 5 are marked with a star (★). This indicates that the mathematical developments of these sections are more difficult and generally use statistical concepts. On the first reading, the beginning student should be concerned with the results of the starred sections and should not labor over the mathematical details until he or she has gained more mathematical expertise, developed in later chapters. The starred sections are included because the results are significant, and it is hoped that these sections will motivate the beginning student to continue to study the exciting subject of communications.

The contents of the book are further subdivided as follows. Chapter 1 provides clear definitions of digital and analog signals as well as giving historical perspective and theoretical limits on the performance of communication systems. Chapter 2 develops the topics of spectra, orthogonal representations, and Fourier theory. Linear system concepts are reviewed. Chapter 3 covers baseband pulse and baseband digital signaling techniques, line codes and their spectra, and the prevention of intersymbol interference. Chapter 4 introduces bandpass communication circuits and develops the theory and practice of bandpass communication systems (amplitude modulation, frequency modulation, etc.). Chapter 5 provides examples of telephone, television, fiber optic, spread spectrum, digital, and satellite communication systems. Summaries of technical standards are given. Chapter 6 develops the mathematical topic of random processes that is needed for analyzing the effects of noise. Chapter 7 describes the performance of digital and analog communication systems that are corrupted by noise. Chapter 8 is concerned with the design of optimum digital receivers that combat the effects of noise. Coding theory is also developed. A summary of useful mathematical techniques and tables is given in Appendix A. Appendix B covers the topic of probability and random variables. (This appendix is a prerequisite for Chapter 6 if the reader does not already have such knowledge.) Appendix C gives standards and terminology that are used in computer communication systems.

This book is written to be applicable to many different course structures. These are summarized in the table.

I appreciate the help of the many persons who contributed to this book. In particular I appreciate the very helpful comments that have been provided by

Course Length[a]	Chapters Covered	Course Title and Comments
Undergraduate		
1 term 1 quarter	1, 2, 3, 4, 5 (partially) 1, 2, 3, 4	Introduction to Digital and Analog Communication Systems (Student background in signals, networks, and statistics not required)
1 term 1 quarter	1, 2 (rapidly), 3, 4, 5 1, 2 (rapidly), 3, 4	Introduction to Digital and Analog Communication Systems (Student background in statistics not required; knowledge of signals and networks required)
1 term	1, Appendix B, 6, 7	Digital and Analog Communication Systems in Noise (A course in probability, random variables, random processes, and applications to communication systems)
1 quarter	1, Appendix B, 6, Secs. 7-1 to 7-6	Digital Communication Systems in Noise
1 quarter	1, Appendix B, 6, Sec. 7-8	Analog Communication Systems in Noise
1 term	1, 2 (rapidly), 3, 4, 7	Digital and Analog Communication Systems (Prior course in random processes required)
Two terms		
1st term	1, 2 (rapidly), 3, 4, 5	Communication I—Introduction to Communication Systems
2nd term	Appendix B, 6, 7	Communication II—Performance of Communication Systems in Noise
Three quarters		
1st quarter	1, 2 (rapidly), 3, 4	Communication I—Introduction to Communication Systems
2nd quarter	5, Appendix B, 6	Communication II—Communication Systems and Noise
3rd quarter	7, 8	Communication III—Performance of Communication Systems and Optimum Digital Receivers
Graduate		
1 term	1, Appendix B, 6, 7, Secs. 8-1 to 8-4	Introduction to Communication Systems (Some undergraduate knowledge of communications required)
1 term	1, 6, 7, 8	Introduction to Communication Systems with Optimum Digital Receivers (Knowledge of probability required)

[a]One term is assumed to be equivalent to 3 class hours per week for a semester system or 4 class hours per week for a quarter system. One quarter is assumed to be 3 class hours per week.

the Macmillan reviewers. For the first edition they were Ray W. Nettleton (Litton Amecon), James A. Cadzow (Arizona State University), Dean T. Davis (The Ohio State University), Jerry D. Gibson (Texas A&M University), and Gerald Lachs (Pennsylvania State University). For the second edition they are

Jeff Burl (University of Calif[illegible]rvine), Donald J. Healy (Georgia Institute of Technology), and Morr[illegible] (Iowa State University).

One reviewer was especially helpful in providing marked manuscript pages with suggestions and corrections. I also appreciate the help of my colleagues at the University of Florida, including Dr. Peyton Z. Peebles. Special thanks to the many University of Florida students who have been most helpful in making suggestions for this second edition. I also thank Lawrence K. Thompson and Charles S. Prewitt for assistance in preparing the solution manual. I am also grateful to the late Dr. T. S. George who taught me a great deal about communications while I was a graduate student under his direction. I am also very appreciative of the help of my wife, Dr. Margaret Couch, who has typed the original and revised manuscripts and proofread the entire book.

Leon W. Couch II
Gainesville, Florida

CONTENTS

LIST OF SYMBOLS xxv

1 INTRODUCTION 1

1-1 Classification of Information Sources 3
1-2 Classification of Communication Systems 3
1-3 Deterministic and Random Waveforms 4
1-4 Organization of This Book 5
1-5 The Block Diagram of a Communication System 6
1-6 Frequency Allocations 8
1-7 Propagation of Electromagnetic Waves 10
1-8 Information Measure 13
Example 1-1 Evaluation of Information and Entropy 14
1-9 Ideal Communication Systems 15
1-10 Preview 16
Problems 17

2 SIGNALS AND NOISE 19

2-1 **Properties of Signals and Noise** 19
PHYSICALLY REALIZABLE WAVEFORMS 20
TIME AVERAGE OPERATOR 22
DC VALUE 23
POWER 23
Example 2-1 Evaluation of Power 24
RMS VALUE AND NORMALIZED POWER 25
ENERGY WAVEFORMS AND POWER WAVEFORMS 26
DECIBEL 27
PHASORS 29

2-2 **Fourier Transform and Spectra** 30
DEFINITION 30
Example 2-2 Spectrum of an Exponential Pulse 32
PROPERTIES OF FOURIER TRANSFORMS 33
PARSEVAL'S THEOREM 34
Example 2-3 Spectrum of a Damped Sinusoid 37
DIRAC DELTA FUNCTION AND UNIT STEP FUNCTION 37
Example 2-4 Spectrum of a Sinusoid 38
RECTANGULAR AND TRIANGULAR PULSES 40
Example 2-5 Spectrum of a Rectangular Pulse 42
Example 2-6 Spectrum of a Triangular Pulse 43
CONVOLUTION 44
Example 2-7 Convolution of a Rectangle with an Exponential 45
Example 2-8 Spectrum of a Triangular Pulse by Convolution 46
Example 2-9 Spectrum of a Switched Sinusoid 46

2-3 **Power Spectral Density and Autocorrelation Function** 47
POWER SPECTRAL DENSITY (PSD) 47
AUTOCORRELATION FUNCTION 49
Example 2-10 PSD of a Sinusoid 49

2-4 **Orthogonal Series Representation of Signals and Noise** 51
ORTHOGONAL FUNCTIONS 51
ORTHOGONAL SERIES 52
VECTOR REPRESENTATIONS OF DIGITAL SIGNALS 54
Example 2-11 Representation of a Digital Signal 55

2-5 **Fourier Series** 57
COMPLEX FOURIER SERIES 57
QUADRATURE FOURIER SERIES 59
POLAR FOURIER SERIES 60
LINE SPECTRA FOR PERIODIC WAVEFORMS 61

Example 2-12 Fourier Coefficients for a Square Wave 63
POWER SPECTRAL DENSITY FOR PERIODIC WAVEFORMS 66
2-6 **Review of Linear Systems** 66
LINEAR TIME-INVARIANT SYSTEMS 66
IMPULSE RESPONSE 67
TRANSFER FUNCTION 68
Example 2-13 RC Low-Pass Filter 69
DISTORTIONLESS TRANSMISSION 71
Example 2-14 Distortion Caused by a Filter 72
2-7 **Bandlimited Signals and Noise** 74
BANDLIMITED WAVEFORMS 74
SAMPLING THEOREM 75
DIMENSIONALITY THEOREM 80
DISCRETE FOURIER TRANSFORM 83
2-8 **Bandwidth of Signals and Noise** 86
Example 2-15 Bandwidths for a BPSK Signal 90
2-9 **Summary** 94
Problems 95

3 **BASEBAND PULSE AND DIGITAL SIGNALING** 105

3-1 **Introduction** 105
3-2 **Pulse Amplitude Modulation** 106
NATURAL SAMPLING (GATING) 108
INSTANTANEOUS SAMPLING (FLAT-TOP PAM) 111
3-3 **Pulse Code Modulation** 114
SAMPLING, QUANTIZING, AND ENCODING 115
EFFECTS OF NOISE AND EYE PATTERNS 119
Example 3-1 Design of a PCM system 121
NONUNIFORM QUANTIZING: μ-LAW AND A-LAW COMPANDING 123
REGENERATIVE REPEATERS AND BIT SYNCHRONIZERS 127
3-4 **Digital Signaling Formats** 130
BINARY LINE CODING 130
POWER SPECTRA OF LINE CODES 133
SPECTRAL EFFICIENCY 138
DIFFERENTIAL CODING 138
MULTILEVEL SIGNALING 140
3-5 **Intersymbol Interference** 142
NYQUIST'S FIRST METHOD (ZERO ISI) 145
RAISED COSINE-ROLLOFF FILTERING 146

Example 3-1 (continued) 148
ZERO-FORCING TRANSVERSAL FILTER EQUALIZERS 151
NYQUIST'S SECOND AND THIRD METHODS FOR CONTROL OF ISI 154
PARTIAL RESPONSE AND DUOBINARY SIGNALING 157

3-6 Differential Pulse Code Modulation **161**

3-7 Delta Modulation **164**
GRANULAR NOISE AND SLOPE OVERLOAD NOISE 167
Example 3-2 Design of a DM system 169
ADAPTIVE DELTA MODULATION AND CONTINUOUSLY VARIABLE SLOPE DELTA MODULATION 170

3-8 Time Division Multiplexing **173**
SYNCHRONOUS AND ASYNCHRONOUS LINES 175
Example 3-3 Design of a Time-Division Multiplexer 176
TDM HIERARCHY 178
THE T1 PCM SYSTEM 180

3-9 Pulse Time Modulation: Pulse Duration Modulation and Pulse Position Modulation **186**

3-10 Summary **191**

Problems **192**

4 BANDPASS SIGNALING TECHNIQUES AND COMPONENTS **202**

4-1 Introduction **202**

4-2 Representation of Bandpass Waveforms and Systems **203**
DEFINITIONS—BASEBAND, BANDPASS, AND MODULATION 203
COMPLEX ENVELOPE REPRESENTATION 204
BANDPASS FILTERING 206
DISTORTIONLESS TRANSMISSION 207
BANDPASS DIMENSIONALITY THEOREM 209
REPRESENTATION OF MODULATED SIGNALS 211
SPECTRUM OF BANDPASS SIGNALS 211
EVALUATION OF POWER 214
Example 4-1 Amplitude-Modulated Signal 215
DIGITAL COMPUTER SIMULATION 216

4-3 Components of Communication Systems **220**
FILTERS 220
AMPLIFIERS 224
LIMITERS 230

MIXERS, UP CONVERTERS, AND DOWN CONVERTERS 231
FREQUENCY MULTIPLIERS 237
ENVELOPE DETECTOR 239
PRODUCT DETECTOR 240
FREQUENCY MODULATION DETECTOR 241
PHASE-LOCKED LOOPS 245

4-4 Transmitters and Receivers **254**
GENERALIZED TRANSMITTERS 254
GENERALIZED RECEIVER: THE SUPERHETERODYNE RECEIVER 256
Example 4-2 AM Broadcast Receiver 257

4-5 Amplitude Modulation and Double-Sideband Suppressed Carrier **259**
AMPLITUDE MODULATION 259
Example 4-3 Power of an AM Signal 262
AM BROADCAST TECHNICAL STANDARDS 265
DOUBLE-SIDEBAND SUPPRESSED CARRIER 267
COSTAS LOOP FOR DEMODULATING A DSC-SC SIGNAL OR A BPSK SIGNAL 267

4-6 Single Sideband and Vestigial Sideband **268**

4-7 Angle Modulation: Phase Modulation and Frequency Modulation **274**
REPRESENTATION OF PM AND FM SIGNALS 274
SPECTRA OF ANGLE MODULATED SIGNALS 279
Example 4-4 Spectrum of a PM or FM signal with Sinusoidal Modulation 279
NARROWBAND ANGLE MODULATION 283
WIDEBAND FREQUENCY MODULATION 284
Example 4-5 Spectrum of WBFM with Triangular Modulation 285
PREEMPHASIS AND DEEMPHASIS IN ANGLE MODULATED SYSTEMS 287
FM BROADCAST TECHNICAL STANDARDS 289
DOLBY AND DBX NOISE REDUCTION SYSTEMS 289

4-8 Summary **292**
Problems **292**

5 BANDPASS COMMUNICATION SYSTEMS **305**

5-1 Introduction **305**
5-2 Digital Communication Systems **306**
BINARY SIGNALING 306

On-Off Keying (OOK) 308
Binary-Phase Shift Keying (BPSK) 309
Frequency Shift Keying (FSK) 311
Example 5-1 Spectrum of the Bell 103 Type FSK Modem 313
MULTILEVEL SIGNALING 319
Quadrature Phase Shift Keying (QPSK) and M-ary Phase Shift Keying (MPSK) 320
Quadrature Amplitude Modulation (QAM) 321
PSD for MPSK and QAM 323
Minimum Shift Keying (MSK) 326

5-3 **Common Carrier Systems** **330**
TIME DIVISION MULTIPLEXING 332
FREQUENCY DIVISION MULTIPLEXING 332
Example 5-2 FM Stereo 332
FDM TELEPHONE HIERARCHY 335
TELEPHONE SYSTEMS 335
CAPACITIES OF COMMON CARRIER LINKS 337

5-4 **Satellite Communication Systems** **340**
TELEVISION SIGNAL TRANSMISSION 343
DATA AND TELEPHONE SIGNAL MULTIPLE ACCESS 344
Example 5-3 Fixed Assigned Multiple Access Mode Using an FDMA Format 347
Example 5-4 SPADE System 347

5-5 **Received Signal-to-Noise Ratio** **351**
SIGNAL POWER RECEIVED 351
THERMAL NOISE SOURCES 354
CHARACTERIZATION OF NOISE SOURCES 355
NOISE CHARACTERIZATION OF LINEAR DEVICES 356
NOISE CHARACTERIZATION OF CASCADED LINEAR DEVICES 361
LINK BUDGET FOR EVALUATION 363
E_b/N_o LINK BUDGET FOR DIGITAL SYSTEMS 365
Example 5-5 Link Budget Evaluation for a TV Receive-Only Terminal for Satellite Signals 366

5-6 **Fiber Optic Systems** **371**
Example 5-6 Link Budget for a Fiber Optic System 371

5-7 **Spread Spectrum Systems** **373**
DIRECT SEQUENCE 374
FREQUENCY HOPPING 378

5-8 **Television** **380**
BLACK-AND-WHITE TELEVISION 380
STEREO SOUND 385
COLOR TELEVISION 386
STANDARDS FOR TV AND CATV SYSTEMS 393

5-9 **Summary** 397
Problems 398

6 RANDOM PROCESSES AND SPECTRAL ANALYSIS 408

6-1 **Some Basic Definitions** 408
RANDOM PROCESSES 408
STATIONARITY AND ERGODICITY 410
Example 6-1 First-Order Stationarity 411
Example 6-2 An Ergodic Random Process 413
CORRELATION FUNCTIONS AND WIDE-SENSE STATIONARITY 414
COMPLEX RANDOM PROCESSES 417
6-2 **Power Spectral Density** 418
DEFINITION 418
WIENER-KHINTCHINE THEOREM 420
PROPERTIES OF PSD 422
Example 6-3 Evaluation of the PSD for a Polar Baseband Signal 423
A GENERAL FORMULA FOR THE PSD OF BASEBAND DIGITAL SIGNALS 427
WHITE NOISE PROCESSES 428
MEASUREMENT OF PSD 429
Analog Techniques 429
Numerical Computation of PSD 429
6-3 **Dc and Rms Values for Random Processes** 430
6-4 **Linear Systems** 432
INPUT–OUTPUT RELATIONSHIPS 432
Example 6-4 Output Autocorrelation and PSD for an RC Low-Pass Filter 435
Example 6-5 Signal-to-Noise Ratio at the Output of an RC LPF 436
6-5 **Bandwidth Measures** 437
EQUIVALENT BANDWIDTH 437
RMS BANDWIDTH 437
Example 6-6 Equivalent Bandwidth and Rms Bandwidth for an RC Low-Pass Filter 439
6-6 **The Gaussian Random Process** 440
PROPERTIES OF GAUSSIAN PROCESSES 441
Example 6-7 A White Gaussian Noise Process 444
6-7 **Bandpass Processes** 444
BANDPASS REPRESENTATIONS 444
PROPERTIES OF WIDE-SENSE STATIONARY BANDPASS PROCESSES 448

Example 6-8 Spectra for the Quadrature Components of White Bandpass Noise 451
Example 6-9 PSD for a BPSK Signal 452
PROOFS OF SOME PROPERTIES 452
Example 6-10 PDF for the Envelope and Phase Functions of a Gaussian Bandpass Process 455

6-8 Matched Filters **458**
GENERAL RESULTS 458
RESULTS FOR WHITE NOISE 460
Example 6-11 Integrate-and-Dump (Matched) Filter 462
CORRELATION PROCESSING 464
Example 6-12 Matched Filter for Detection of a BPSK Signal 465
TRANSVERSAL MATCHED FILTER 467

6-9 Summary **469**
Problems **471**

7 PERFORMANCE OF COMMUNICATION SYSTEMS CORRUPTED BY NOISE **481**

7-1 Error Probabilities for Binary Signaling **482**
GENERAL RESULTS 482
RESULTS FOR GAUSSIAN NOISE 485
RESULTS FOR WHITE GAUSSIAN NOISE AND MATCHED FILTER RECEPTION 487
RESULTS FOR COLORED GAUSSIAN NOISE AND MATCHED FILTER RECEPTION 488

7-2 Performance of Baseband Binary Systems **489**
UNIPOLAR SIGNALING 489
POLAR SIGNALING 490

7-3 Coherent Detection of Bandpass Binary Signals **492**
ON-OFF KEYING 492
BINARY PHASE SHIFT KEYING 494
FREQUENCY SHIFT KEYING 495

7-4 Noncoherent Detection of Bandpass Binary Signals **499**
ON-OFF KEYING 499
FREQUENCY SHIFT KEYING 503
DIFFERENTIAL PHASE SHIFT KEYING 505

7-5 Quadrature Phase Shift Keying and Minimum Shift Keying **507**

7-6 Comparison of Digital Signaling Systems **510**
BIT ERROR RATE AND BANDWIDTH 510
SYNCHRONIZATION 511

7-7 Output Signal-to-Noise Ratio for PCM Systems 512

7-8 Output Signal-to-Noise Ratios for Analog Systems 518

BASEBAND SYSTEMS 518

AM SYSTEMS WITH PRODUCT DETECTION 519

AM SYSTEMS WITH ENVELOPE DETECTION 520

DSB–SC SYSTEMS 522

SSB SYSTEMS 522

PM SYSTEMS 523

FM SYSTEMS 527

FM SYSTEMS WITH THRESHOLD EXTENSION 529

FM SYSTEMS WITH DEEMPHASIS 532

7-9 Comparison of Analog Signaling Systems 535

IDEAL SYSTEM PERFORMANCE 536

7-10 Summary 537

Problems 537

8 OPTIMUM DIGITAL RECEIVERS 547

8-1 Introduction 547

8-2 Optimization Criteria 548

BAYES CRITERION 548

Example 8-1 Bayes Criterion for a Binary Source 549

MINIMAX CRITERION 551

NEYMAN–PEARSON CRITERION 551

MAXIMUM A POSTERIORI CRITERION 551

8-3 *N*-Dimensional Vector Space 553

SIGNAL REPRESENTATION 553

GRAM–SCHMIDT ORTHOGONALIZATION PROCEDURE 557

Example 8-2 Vectors for a Rectangular Waveform Set 559

VECTOR CHANNELS AND RELEVANT NOISE 562

PDF FOR THE NOISE VECTOR 564

8-4 MAP Receiver 566

DECISION RULE FOR THE ADDITIVE WHITE GAUSSIAN NOISE CHANNEL 566

RECEIVER STRUCTURES 568

8-5 Probability of Error for MAP Receivers 574

Example 8-3 Binary Signaling 574

Example 8-4 Quadrature Amplitude Modulation Signaling 576

Example 8-5 Pulse Code Modulation Signaling 578

UNION BOUND 580

Example 8-6 Quantized Pulse Position Modulation Signaling 581
8-6 Random Phase and Fading Channels **583**
DOUBLE-SIDEBAND TRANSMISSION WITH RANDOM PHASE 583
PROBABILITY OF ERROR FOR ORTHOGONAL BINARY RANDOM-PHASE SIGNALING 591
Example 8-7 FSK Orthogonal Signaling 593
FADING CHANNELS 593
Example 8-8 Probability of Error for a Fading Channel with FSK Signaling 595
8-7 Bit Error Rate for DPSK Signaling **597**
8-8 Coding **601**
BLOCK CODES 603
Example 8-9 Hamming Code 605
CONVOLUTIONAL CODES 605
CODE PERFORMANCE 609
8-9 Summary **611**
Problems **612**

APPENDIX A MATHEMATICAL TECHNIQUES, IDENTITIES, AND TABLES **618**
A-1 Trigonometry **618**
A-2 Differential Calculus **619**
A-3 Indeterminate Forms **621**
A-4 Integral Calculus **621**
A-5 Integral Tables **622**
A-6 Series Expansions **625**
A-7 Hilbert Transform Pairs **627**
A-8 The Dirac Delta Function **627**
A-9 Tabulation of $\mathrm{Sa}(x) = (\sin x)/x$ **629**
A-10 Tabulation of $Q(z)$ **630**

APPENDIX B PROBABILITY AND RANDOM VARIABLES **632**
B-1 Introduction **632**
B-2 Sets **633**
B-3 Probability and Relative Frequency **634**
B-4 Random Variables **637**
B-5 Cumulative Distribution Functions and Probability Density Functions **638**
B-6 Ensemble Average and Moments **644**

B-7 Examples of Important Distributions 647
B-8 Functional Transformations of Random Variables 656
B-9 Multivariate Statistics 661
Problems 670

APPENDIX C STANDARDS AND TERMINOLOGY FOR COMPUTER COMMUNICATIONS 676
C-1 Codes 676
C-2 DTE/DCE Interface Standards 681
C-3 The ISO OSI Network Model 687
C-4 Data Link Control Protocols 689
C-5 Telephone Line Standards 691
C-6 Modem Standards 692
C-7 Brief Computer Communications Glossary 692

REFERENCES 697

ANSWERS TO SELECTED PROBLEMS 704

INDEX 713

LIST OF SYMBOLS

There are not enough symbols in the English and Greek alphabets to allow the use of each letter only once. Consequently, some symbols may be used to denote more than one entity, but their use should be clear from the context. Furthermore, the symbols are chosen to be generally the same as those used in the associated mathematical discipline. For example, in the context of complex variables, x denotes the real part of a complex number (i.e., $c = x + jy$), whereas in the context of statistics x might denote a random variable.

Symbols

a_n	A constant
a_n	Quadrature Fourier series coefficient
A_c	Level of modulated signal of carrier frequency f_c
A_e	Effective area of an antenna
b_n	Quadrature Fourier series coefficient

B	Baseband bandwidth
B_p	Bandpass filter bandwidth
B_T	Transmission (bandpass) bandwidth
c	A complex number where $c = x + jy$
c	A constant
c_n	Complex Fourier series coefficient
C	Channel capacity
C	Capacitance
C	Cost matrix
$\overline{C}$	Average cost
°C	Degrees Celsius
dB	Decibel
D	Dimensions/sec ($D = N/T_0$) or baud rate
D_f	Frequency modulation gain constant
D_n	Polar Fourier series coefficient
D_p	Phase modulation gain constant
e	Error
e	The natural number, 2.7183
E	Modulation efficiency
E	Energy
$\mathcal{E}(f)$	Energy spectral density (ESD)
E_b/N_o	Energy per bit/noise power spectral density ratio
f	Frequency (Hz)

$f(x)$	Probability density function (pdf)
f_c	Carrier frequency
f_i	Instantaneous frequency
f_o	A (frequency) constant; the fundamental frequency of a periodic waveform
f_s	Sampling frequency
F	Noise figure
$F(a)$	Cumulative distribution function (cdf)
$g(t)$	Complex envelope
$\tilde{g}(t)$	Corrupted complex envelope
G	Power gain
$G(f)$	Power transfer function
h	Planck's constant, 6.2×10^{-34} joule-sec
$h(t)$	Impulse response of a linear network
$h(x)$	Mapping function of x into $h(x)$
H	Entropy
$H(f)$	Transfer function of a linear network
i	An integer
I_j	Information in the jth message
j	The imaginary number $\sqrt{-1}$
k	Boltzmann's constant, 1.38×10^{-23} joule/K
k	An integer
$k(t)$	Complex impulse response of a bandpass network
K	Number of bits in a binary word that represents a digital message

K	Degrees Kelvin (°C + 273)
l	An integer
ℓ	Number of bits per dimension
L	Inductance
L	Number of levels permitted
m	An integer
m	Mean value
$m(t)$	Message (modulation) waveform
$\tilde{m}(t)$	Corrupted (noisy received) message
M	An integer
M	Number of messages permitted
n	An integer
n	Number of bits in message
$n(t)$	Noise waveform
N	An integer
N	Number of dimensions used to represent a digital message
N	Noise power
N_o	Level of the power spectral density of white noise
$p(t)$	An absolutely time-limited pulse waveform
$p(t)$	Instantaneous power
$p(m)$	Probability density function of frequency modulation
P	Average power
P_e	Probability of bit error

$P(C)$	Probability of correct decision
$P(E)$	Probability of message error
$\mathcal{P}(f)$	Power spectral density (PSD)
$Q(z)$	Integral of a Gaussian function
$Q(x_k)$	Quantized value of the kth sample value, x_k
$r(t)$	Received signal plus noise
R	Data rate (bits/sec)
R	Resistance
$R(t)$	Real envelope
$R(\tau)$	Autocorrelation function
$s(t)$	Signal
$\tilde{s}(t)$	Corrupted signal
S/N	Signal power/noise power ratio
t	Time
T	A time interval
T	Absolute temperature (Kelvin)
T_b	Bit period
T_e	Effective input-noise temperature
T_o	Duration of a transmitted symbol or message
T_o	Period of a periodic waveform
T_o	Standard room temperature (290 K)
T_s	Sampling period
u_{11}	Covariance

$v(t)$	A voltage waveform
$v(t)$	A bandpass waveform or a bandpass random process
$w(t)$	A waveform
$W(f)$	Spectrum (Fourier transform) of $w(t)$
x	An input
x	A random variable
x	Real part of a complex function or a complex constant
$x(t)$	A random process
y	An output
y	An output random variable
y	Imaginary part of a complex function or a complex constant
$y(t)$	A random process
α	A constant
β	A constant
β_f	Frequency modulation index
β_p	Phase modulation index
δ	Step size of delta modulation
$\delta(t)$	Impulse (Dirac delta function)
ΔF	Peak frequency deviation
$\Delta\theta$	Peak phase deviation
ϵ	A constant
ϵ	Error
η	Spectral efficiency [(bits/sec)/Hz]

$\theta(t)$	Phase waveform
λ	Dummy variable of integration
λ	Wavelength
$\Lambda(r)$	Likelihood ratio
π	3.14159
ρ	Correlation coefficient
σ	Standard deviation
τ	Independent variable of autocorrelation function
τ	Pulse width
$\varphi_j(t)$	Orthogonal function
ϕ_n	Polar Fourier series coefficient
ω_c	Radian carrier frequency, $2\pi f_c$
$\equiv$	Mathematical equivalents
$\triangleq$	Mathematical definition of a symbol

Defined functions

$J_n(\cdot)$	Bessel function of the first kind *n*th order
$\ln(\cdot)$	Natural logarithm
$\log(\cdot)$	Base 10 logarithm
$\log_2(\cdot)$	Base 2 logarithm
$Q(z)$	Integral of a Gaussian probability density function
$\mathrm{Sa}(z)$	$(\sin z)/z$
$u(\cdot)$	Unit step function

$\Lambda(\cdot)$ Triangle function

$\Pi(\cdot)$ Rectangle function

Operator notation

$\mathrm{Im}\{\cdot\}$ Imaginary part of

$\mathrm{Re}\{\cdot\}$ Real part of

$\overline{[\cdot]}$ Ensemble average

$\langle[\cdot]\rangle$ Time average

$[\cdot] * [\cdot]$ Convolution

$[\cdot]^*$ Conjugate

$\angle[\cdot]$ Angle operator

$|[\cdot]|$ Absolute value

$[\hat{\cdot}]$ Hilbert transform

$\mathscr{F}[\cdot]$ Fourier transform

$\mathscr{L}[\cdot]$ Laplace transform

$[\cdot] \bullet [\cdot]$ Dot product

CHAPTER 4

Bandpass Signaling Techniques and Components

4-1 INTRODUCTION

This chapter is concerned with *bandpass* signaling techniques. These are applicable to both digital and analog communication systems. As indicated in Chapter 1, the bandpass communication signal is obtained by modulating the baseband signal onto a carrier. The baseband signal might be an analog signal, such as that obtained directly from a microphone, or it might be digital, such as a PCM signal as discussed in Chapter 3. Here the classical bandpass signaling techniques of amplitude modulation, single-sideband, and angle modulation will be studied in detail. Classical modulation theory is directly applicable to the understanding of digital bandpass signaling techniques that are studied in Chapter 5.

For a better understanding of the implementation of communication systems, a description of communication component blocks such as filters, amplifiers, up-and-down converters, and detectors is covered in Sec. 4-3. (This section may be skipped if this material has been covered in electronics courses.)

Block diagrams for the various types of transmitters and receivers will be illustrated and analyzed. In addition to the theory, practical aspects of transmitter and receiver design will be emphasized.

First, we will study the mathematical basis of modulation theory.

4-2 REPRESENTATION OF BANDPASS WAVEFORMS AND SYSTEMS

What is a general representation for bandpass digital and analog signals? How do we represent a modulated signal? How do we represent bandpass noise? These are some of the questions that are answered in this section.

Definitions: Baseband, Bandpass, and Modulation

Definition. A *baseband* waveform has a spectral magnitude that is nonzero for frequencies in the vicinity of the origin (i.e., $f = 0$) and negligible elsewhere.

Definition. A *bandpass* waveform has a spectral magnitude that is nonzero for frequencies in some band concentrated about a frequency $f = \pm f_c$, where $f_c >> 0$. The spectral magnitude is negligible elsewhere. f_c is called the *carrier frequency*.

For bandpass waveforms the value of f_c may be arbitrarily assigned for mathematical convenience in *some* problems. In others, namely, modulation problems, f_c is the frequency of an oscillatory signal in the transmitter circuit and is the assigned frequency of the transmitter, such as, for example, 850 kHz for an AM broadcasting station.

In communication problems, the information source signal is usually a baseband signal: for example, a transistor-transistor logic (TTL) waveform from a digital circuit, an audio signal from a microphone, or a video signal from a television camera. As described in Chapter 1, the communication engineer has the job of building a system that will transfer the information in this source signal $m(t)$ to the desired destination. As shown in Fig. 4-1, this usually requires the use of a bandpass signal, $s(t)$, which has a bandpass spectrum that is con-

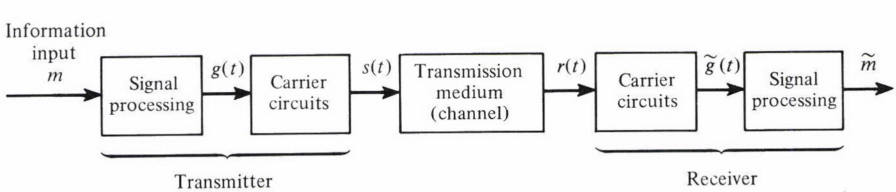

FIGURE 4-1 A communication system.

centrated at $\pm f_c$ so that it will propagate across the communication channel (either a softwire or a hardwire channel).

Definition. *Modulation* is the process of encoding the source information onto a bandpass signal with a carrier frequency f_c. This bandpass signal is called the *modulated* signal $s(t)$, and the baseband source signal is called the *modulating* signal $m(t)$.

Examples of exactly how modulation is accomplished are given later in this chapter. The definition indicates that modulation may be visualized as a mapping operation that maps the source information onto the bandpass signal $s(t)$ that will be transmitted over the channel.

As the modulated signal passes through the channel, noise corrupts it. The result is a bandpass signal-plus-noise waveform that is available at the receiver input, $r(t)$ (see Fig. 4-1). The receiver has the job of trying to recover the information that was sent from the source. $\tilde{m}$ denotes the corrupted version of m.

Complex Envelope Representation

It is very interesting that *all* bandpass waveforms, whether they arise due to a modulated signal, interfering signals, or noise, may be represented in a convenient form given by the following theorem. $v(t)$ will be used to denote the bandpass waveform canonically; specifically, it can represent the signal when $s(t) \equiv v(t)$, the noise when $n(t) \equiv v(t)$, the filtered signal plus noise at the channel output when $r(t) \equiv v(t)$, or any other type of bandpass waveform.

Theorem. Any bandpass waveform may be represented by

$$v(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} \tag{4-1a}$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part of $\{\cdot\}$, $g(t)$ is called the *complex envelope* of $v(t)$, and f_c is the associated carrier frequency, $\omega_c = 2\pi f_c$. Furthermore, two other equivalent representations are†

$$v(t) = R(t)\cos[\omega_c t + \theta(t)] \tag{4-1b}$$

and

$$v(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t \tag{4-1c}$$

where

$$\begin{aligned} g(t) &= x(t) + jy(t) = |g(t)|e^{j\underline{/g(t)}} \\ &\equiv R(t)e^{j\theta(t)} \end{aligned} \tag{4-2}$$

†The symbol $\equiv$ denotes an equivalence and the symbol $\triangleq$ denotes a definition.

and

$$x(t) = \mathrm{Re}\{g(t)\} \equiv R(t)\cos\theta(t) \tag{4-3a}$$

$$y(t) = \mathrm{Im}\{g(t)\} \equiv R(t)\sin\theta(t) \tag{4-3b}$$

$$R(t) \triangleq |g(t)| \equiv \sqrt{x^2(t) + y^2(t)} \tag{4-4a}$$

$$\theta(t) \triangleq \underline{/g(t)} = \tan^{-1}\left(\frac{y(t)}{x(t)}\right) \tag{4-4b}$$

The waveforms $g(t)$, $x(t)$, $y(t)$, $R(t)$, and $\theta(t)$ are *all* baseband waveforms and, *except* for $g(t)$, they are all real waveforms. $R(t)$ is a nonnegative real waveform.

Proof. Any physical waveform (it does *not* have to be periodic) may be represented over all time, $T_o \to \infty$, by the complex Fourier series:

$$v(t) = \sum_{n=-\infty}^{n=\infty} c_n e^{jn\omega_o t}, \qquad \omega_o = 2\pi/T_o \tag{4-5}$$

Furthermore, since the physical waveform is real $c_{-n} = c_n^*$ and using $\mathrm{Re}\{\cdot\} = \frac{1}{2}\{\cdot\} + \frac{1}{2}\{\cdot\}^*$,

$$v(t) = \mathrm{Re}\left\{c_o + 2\sum_{n=1}^{\infty} c_n e^{jn\omega_o t}\right\} \tag{4-6}$$

Introducing the arbitrary parameter f_c, this becomes†

$$v(t) = \mathrm{Re}\left\{\left(c_o e^{-j\omega_c t} + 2\sum_{n=1}^{n=\infty} c_n e^{j(n\omega_o - \omega_c)t}\right) e^{j\omega_c t}\right\} \tag{4-7}$$

so that (4-1*a*) follows where

$$g(t) \equiv c_o e^{-j\omega_c t} + 2\sum_{n=1}^{\infty} c_n e^{j(n\omega_o - \omega_c)t} \tag{4-8}$$

Since $v(t)$ is a bandpass waveform with nonzero spectrum concentrated near $f = f_c$, the Fourier coefficients c_n are nonzero only for values of n in the range $\pm nf_o \approx f_c$. Therefore, from (4-8) $g(t)$ has a spectrum that is concentrated near $f = 0$. That is, $g(t)$ is a *baseband* waveform. It is also obvious from (4-8) that $g(t)$ may be a complex function of time. Representing the complex envelope in terms of two real functions in Cartesian coordinates, we have

$$g(t) \equiv x(t) + jy(t)$$

†Since the frequencies involved in the argument of $\mathrm{Re}\{\cdot\}$ are all positive, it can be shown that the complex function $c_o + 2\sum_{n=1}^{\infty} c_n e^{jn\omega_o t}$ is analytic in the upper-half complex t plane. Many interesting properties result because this function is an analytic function of a complex variable.

where $x(t) = \text{Re}\{g(t)\}$ and $y(t) = \text{Im}\{g(t)\}$. $x(t)$ is said to be the *in phase modulation* associated with $v(t)$ and $y(t)$ is said to be the *quadrature modulation* associated with $v(t)$. Alternatively, the polar form of $g(t)$, represented by $R(t)$ and $\theta(t)$, is given by (4-2), where the identities between Cartesian and polar coordinates are given by (4-3) and (4-4). $R(t)$ and $\theta(t)$ are real waveforms and, in addition, $R(t)$ is always nonnegative. $R(t)$ is said to be the *amplitude modulation* (AM) on $v(t)$ and $\theta(t)$ is said to be the *phase modulation* (PM) on $v(t)$. It is also realized that if $v(t)$ is a deterministic waveform, $x(t)$, $y(t)$, $R(t)$, and $\theta(t)$ are also deterministic. If $v(t)$ is stochastic, for example representing bandpass noise, $x(t)$, $y(t)$, $R(t)$, and $\theta(t)$ are stochastic baseband processes. Thus, in general, bandpass noise includes both AM, $R(t)$, and PM, $\theta(t)$, noise components. This will be discussed further in Chapter 6. The usefulness of the complex envelope representation for bandpass waveforms cannot be overemphasized. In modern communication systems, the bandpass signal is often partitioned into two channels, one for $x(t)$ called the I (in-phase) channel and one for $y(t)$ called the Q (quadrature-phase) channel.

Bandpass Filtering

In Sec. 2-6 the general transfer function technique was described for the treatment of linear filter problems. Now a shortcut technique will be developed for bandpass filters. It is emphasized that this technique is applicable only for bandpass filters and waveforms. Here the complex envelope can also be used to describe bandpass filter characteristics. Since the impulse response of a bandpass filter $h(t)$ is a (deterministic) bandpass waveform, it may be represented by a complex envelope $k(t)$ as shown in Fig. 4-2.

Theorem. Referring to Fig. 4-2, the complex envelope out of a bandpass filter is given by

$$g_2(t) = \tfrac{1}{2} g_1(t) * k(t) \tag{4-9}$$

where $g_1(t)$ is the complex envelope of the input and $k(t)$ is the complex envelope of the impulse response. It also follows that

$$G_2(f) = \tfrac{1}{2} G_1(f) K(f) \tag{4-10}$$

This theorem may be proved by using the properties of linear systems (Sec. 2-6) and is given as a homework problem at the end of this chapter.

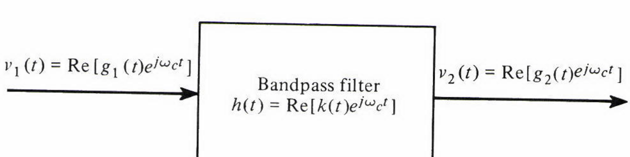

FIGURE 4-2 Filtering bandpass waveforms.

It is realized that the linear bandpass filter can produce variations in the phase modulation at the output, $\theta_2(t)$, where $\theta_2(t) = \angle g_2(t)$, as a function of the amplitude modulation on the input complex envelope, $R_1(t)$, where $R_1(t) = |g_1(t)|$. This is called *AM to PM conversion*. Similarly, the filter can also cause variations on the AM at the output, $R_2(t)$, due to the PM on the input, $\theta_1(t)$. This is called *PM to AM conversion*.

Distortionless Transmission

In Sec. 2-6 the general conditions were found for distortionless transmission. For linear *bandpass filters* (channels), a less restrictive set of conditions will now be shown to be satisfactory. For distortionless transmission of bandpass signals, the channel transfer function, $H(f) = |H(f)|e^{j\theta(f)}$, needs to satisfy the following requirements:

1. The amplitude response is constant. That is,

$$|H(f)| = A \tag{4-11a}$$

   where A is a positive (real) constant.
2. The derivative of the phase response is a constant. That is,

$$-\frac{1}{2\pi}\frac{d\theta(f)}{df} = T_g \tag{4-11b}$$

   where T_g is a constant called the *group delay* and $\theta(f) = \angle H(f)$.

This is illustrated in Fig. 4-3. Note that (4-11*a*) is identical to the general requirement of (2-150a), but (4-11b) is less restrictive than (2-105b). That is, if (2-150b) is satisfied, (4-11b) is satisfied where $T_d = T_g$; however, if (4-11b) is satisfied, (2-150b) is not necessarily satisfied because the integral of (4-11b) is

$$\theta(f) = -2\pi f T_g + \theta_o \tag{4-12}$$

where θ_o is a phase shift constant, as shown in Fig. 4-3b. If θ_o happens to be nonzero, (2-150b) is not satisfied.

Now it will be shown that (4-11a) and (4-11b) are sufficient requirements for distortionless transmission of bandpass signals. From (4-11a) and (4-12) the channel (or filter) transfer function is

$$H(f) = Ae^{j(-2\pi f T_g + \theta_o)} = (Ae^{j\theta_o})e^{-j2\pi f T_g} \tag{4-13}$$

over the bandpass of the signal. If the input to the bandpass channel is represented by

$$v_1(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

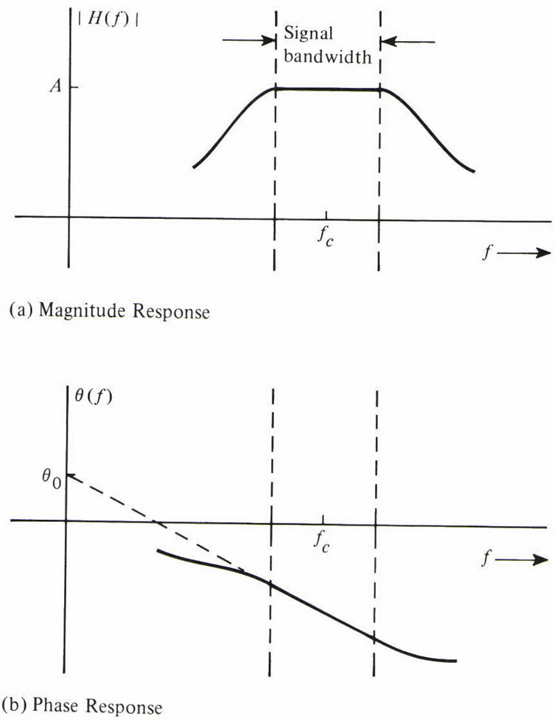

FIGURE 4-3 Transfer characteristics of a distortionless bandpass channel.

then, using (4-13) and realizing that $e^{-j2\pi f T_g}$ causes a delay of T_g, the output of the channel is

$$v_2(t) = Ax(t - T_g)\cos[\omega_c(t - T_g) + \theta_o] - Ay(t - T_g)\sin[\omega_c(t - T_g) + \theta_o]$$

Using (4-12), this becomes

$$v_2(t) = Ax(t - T_g)\cos[\omega_c t + \theta(f_c)] - Ay(t - T_g)\sin[\omega_c t + \theta(f_c)]$$

Furthermore, using (2-150b), the phase shift at $f = f_c$ can be expressed in terms of the phase delay, T_d, since $\theta(f_c) = -2\pi f_o T_d$. (Actually, this is why T_d is called the *phase delay*.) Thus the output bandpass signal reduces to

$$v_2(t) = Ax(t - T_g)\cos[\omega_c(t - T_d) - Ay(t - T_g)\sin[\omega_c(t - T_d)] \quad (4\text{-}14)$$

where T_g is the group delay and T_d is the phase delay.

Equation (4-14) shows that the x and y components of the input complex envelope have been delayed by T_g, the group (or information) delay; and the quadrature sinusoidal carriers have been delayed by T_d, the phase delay. This gives an undistorted received transmission since the input complex envelope, $g(t) = x(t) + jy(t)$, has just been multiplied by the gain factor A and delayed by the group delay, T_g. Note that in this application the carrier (phase) delay, T_d, is not equal to the envelope (group) delay, T_g, unless θ_o happens to be zero. This is also verified by looking at Fig. 4-3b.

In summary, the general requirement for distortionless transmission of either baseband or bandpass signals is given by (2-150). However, the linear phase requirement of (2-150b) is overly restrictive for the case of bandpass signaling. From (4-11) for the bandpass case, it is only necessary to have a transfer function with *constant amplitude* and a *constant phase derivative* over the bandwidth of the signal.

Bandpass Dimensionality Theorem

Computer simulation is often used to analyze the performance of complicated communication systems. This is especially true when coding schemes are used. This requires that the RF signal and noise for the system under test be sampled so that the data can be obtained for processing by digital computer simulation. If the sampling is carried out at the Nyquist rate or larger ($f_s \geq 2B$, where B is the highest frequency involved in the spectrum of the RF signal), the sampling rate would be ridiculous. For example, consider a satellite communication system with a carrier frequency of $f_c = 6$ GHz. The sampling rate required would be at least 12 GHz. Fortunately, for the signals of this type (bandpass signals), it can be shown that the sampling rate depends only on the *bandwidth* of the signal, not on the absolute frequencies involved. This is equivalent to saying that we can reproduce the signal from samples of the complex envelope.

Bandpass Sampling Theorem. If a (real) bandpass waveform has a nonzero spectrum only over the frequency interval $f_1 < |f| < f_2$ where the transmission bandwidth B_T is taken to be the absolute bandwidth given by $B_T = f_2 - f_1$, then the waveform may be reproduced from sample values, if the sampling rate is

$$f_s \geq 2B_T \tag{4-15}$$

This theorem may be demonstrated by using the quadrature bandpass representation

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t) \tag{4-16}$$

Let f_c be the center of the bandpass so that $f_c = (f_2 + f_1)/2$. Then, from (4-8) it is seen that $x(t)$ and $y(t)$ are both baseband signals and absolutely bandlimited to $B = B_T/2$. From the baseband sampling theorem the sampling rate required to represent these baseband signals is $f_b \geq 2B = B_T$. Equation (4-16) becomes

$$v(t) = \sum_{n=-\infty}^{n=\infty} \left[x\left(\frac{n}{f_b}\right) \cos\omega_c t - y\left(\frac{n}{f_b}\right) \sin\omega_c t \right] \cdot \frac{\sin\{\pi f_b[t - (n/f_b)]\}}{\pi f_b[t - (n/f_b)]} \tag{4-17}$$

For the general case, where the $x(n/f_b)$ and $y(n/f_b)$ samples are independent, two samples are obtained for each value of n so that the overall sampling rate for $v(t)$ is $f_s = 2f_b \geq 2B_T$. This is the bandpass sampling frequency requirement of (4-15). In most engineering applications, $f_c >> B_T$. Thus the x and y samples can be obtained by sampling $v(t)$ at $t \approx (n/f_b)$, but adjusting t slightly, so that $\cos \omega_c t = 1$ and $\sin \omega_c t = 1$ at the *exact* sampling time for x and y, respectively. That is, for $t \approx n/f_s$, $v(n/f_b) = x(n/f_b)$ when $\cos \omega_c t = 1$ (i.e., $\sin \omega_c t = 0$) and $v(n/f_b) = y(n/f_b) = y(n/f_b)$ when $\sin \omega_c t = 1$ (i.e., $\cos \omega_c t = 0$). If f_c is not large enough for the x and y samples to be obtained directly from $v(t)$, then $x(t)$ and $y(t)$ can first be obtained by the use of two quadrature product detectors as described by (4-71). The $x(t)$ and $y(t)$ baseband signals can then be individually sampled at a rate of f_b, and it is seen that the overall equivalent sampling rate is still $f_s = 2f_b \geq 2B_T$.

In the application of this theorem it is assumed that the bandpass signal $v(t)$ is reconstructed by use of (4-17). This implies that *nonuniformly* spaced time samples of $v(t)$ are used since the samples are taken in pairs (for the x and y components) instead of being uniformly spaced T_s sec apart. *Uniformly* spaced samples of $v(t)$ itself can be used with a minimum sampling frequency of $2B_T$, *provided* that either f_1 or f_2 is a harmonic of f_s [Taub and Schilling, 1971 or 1986]. Otherwise, a minimum sampling frequency larger than $2B_T$ but not larger than $4B_T$ is required [Haykin, 1983; Peebles, 1976; Taub and Schilling, 1986]. Referring to (2-173), this phenomenon occurs with impulse sampling because f_s needs to be selected so that there is no spectral overlap in the $f_1 < f < f_2$ band when the bandpass spectrum is translated to harmonics of f_s.

Bandpass Dimensionality Theorem. Assume that a bandpass waveform has a nonzero spectrum only over the frequency interval $f_1 < |f| < f_2$ where the transmission bandwidth B_T is taken to be the absolute bandwidth given by $B_T = f_2 - f_1$ and $B_T << f_1$. The waveform may be completely specified over a T_o-sec interval by

$$N = 2B_T T_o \tag{4-18}$$

independent pieces of information. N is said to be the number of dimensions required to specify the waveform.

The bandpass dimensionality theorem is similar to the dimensionality theorem discussed earlier, (2-174), and may be demonstrated by the use of the complex Fourier series to represent the bandpass waveform over a T_o-sec interval. The number of nonzero Fourier coefficients required to produce a line spectrum over the B_T hertz frequency interval is B_T/f_o since f_o is the frequency spacing between adjacent lines. Since $c_n = |c_n|e^{j\angle c_n}$ and $c_{-n} = c_n^*$, the number of independent (real value) samples is $2B_T/f_o = 2B_T T_o$.

As mentioned earlier, computer simulation is often used to analyze communication systems. The bandpass dimensionality theorem tells us that a bandpass signal B_T hertz wide can be represented over a T_o-sec interval provided that at least $N = 2B_T T_o$ samples are used.

Representation of Modulated Signals

As defined earlier, modulation is the process of encoding the source information (modulation) $m(t)$ into a bandpass signal (modulated signal) $s(t)$. Consequently, the modulated signal is just a special application of the bandpass representation. The *modulated signal* is given by

$$s(t) = \text{Re}\{g(t)e^{j\omega_c t}\} \tag{4-19}$$

where $\omega_c = 2\pi f_c$ and f_c is the carrier frequency. The complex envelope $g(t)$ is a function of the modulating signal $m(t)$. That is,

$$g(t) = g[m(t)] \tag{4-20}$$

Thus $g[\cdot]$ performs a mapping operation on $m(t)$. This was illustrated in Fig. 4-1.

In Table 4-1, examples of the mapping function $g[m]$ are given for various types of modulation: amplitude modulation (AM), double-sideband suppressed carrier (DSB-SC), phase modulation (PM), frequency modulation (FM), single-sideband AM suppressed carrier (SSB-AM-SC), single-sideband PM (SSB-PM), single-sideband FM (SSB-FM), single-sideband envelope detectable (SSB-EV), single-sideband square law detectable (SSB-SQ), and quadrature modulation (QM). The parameter f_c is the assigned carrier frequency. These modulated signals are discussed in more detail in Secs. 4-5, 4-6, and 4-7. Obviously, it is possible to use other $g[m]$ functions that are not listed in Table 4-1. The question is: Are they useful? $g[m]$ functions are desired that are easy to implement and that will give desirable spectral and noise (reduction) properties. Furthermore, in the receiver the inverse function $m[g]$ is required. This needs to exist, to be easily implemented, and to suppress as much noise as possible so that $m(t)$ can be recovered with as little corruption as possible.

Spectrum of Bandpass Signals

The spectrum of a bandpass signal is directly related to the spectrum of its complex envelope.

Theorem. If a bandpass waveform is represented by

$$v(t) = \text{Re}\{g(t)e^{j\omega_c t}\} \tag{4-21}$$

then the spectrum of the bandpass waveform is

$$V(f) = \tfrac{1}{2}[G(f - f_c) + G^*(-f - f_c)] \tag{4-22}$$

and the PSD of the waveform is

$$\mathcal{P}_v(f) = \tfrac{1}{4}[\mathcal{P}_g(f - f_c) + \mathcal{P}_g(-f - f_c)] \tag{4-23}$$

where $G(f) = \mathcal{F}[g(t)]$ and $\mathcal{P}_g(f)$ is the PSD of $g(t)$.

Table 4-1 Complex Envelope Functions for Various Types of Modulation

Type of Modulation	Mapping Functions $g[m]$	Corresponding Quadrature Modulation $x(t)$	$y(t)$
AM	$1 + m(t)$	$1 + m(t)$	0
DSB-SC	$m(t)$	$m(t)$	0
PM	$e^{jD_p m(t)}$	$\cos[D_p m(t)]$	$\sin[D_p m(t)]$
FM	$e^{jD_f \int_{-\infty}^{t} m(\sigma)d\sigma}$	$\cos\left[D_f \int_{-\infty}^{t} m(\sigma)d\sigma\right]$	$\sin\left[D_f \int_{-\infty}^{t} m(\sigma)d\sigma\right]$
SSB-AM-SC[a]	$m(t) \pm j\hat{m}(t)$	$m(t)$	$\pm\hat{m}(t)$
SSB-PM[a]	$e^{jD_p[m(t) \pm j\hat{m}(t)]}$	$e^{\mp D_p \hat{m}(t)}\cos[D_p m(t)]$	$e^{\mp D_p \hat{m}(t)}\sin[D_p m(t)]$
SSB-FM[a]	$e^{jD_f \int_{-\infty}^{t}[m(\sigma) \pm j\hat{m}(\sigma)]d\sigma}$	$e^{\mp D_f \int_{-\infty}^{t}\hat{m}(\sigma)d\sigma} \cos\left[D_f \int_{-\infty}^{t} m(\sigma)d\sigma\right]$	$e^{\mp D_f \int_{-\infty}^{t}\hat{m}(\sigma)d\sigma} \sin\left[D_f \int_{-\infty}^{t} m(\sigma)d\sigma\right]$
SSB-EV[a]	$e^{\{\ln[1+m(t)] \pm j\hat{\ln}[1+m(t)]\}}$	$[1 + m(t)] \cos\{\hat{\ln}[1 + m(t)]\}$	$\pm[1 + m(t)] \sin\{\hat{\ln}[1 + m(t)]\}$
SSB-SQ[a]	$e^{(1/2)\{\ln[1+m(t)] \pm j\hat{\ln}[1+m(t)]\}}$	$\sqrt{1 + m(t)} \cos\{\frac{1}{2}\hat{\ln}[1 + m(t)]\}$	$\pm\sqrt{1 + m(t)} \sin\{\frac{1}{2}\hat{\ln}[1 + m(t)]\}$
QM	$m_1(t) + jm_2(t)$	$m_1(t)$	$m_2(t)$

L = linear; NL = nonlinear; $[\hat{\cdot}]$ is the Hilbert transform (i.e., $-90°$ phase-shifted version) of $[\cdot]$ (see Sec. 4-6 and Sec. A-7, Appendix A).

[a]Use upper signs for upper sideband signals and lower signs for lower sideband signals.

[b]In the strict sense, AM signals are not linear because the carrier term does not satisfy the linearity (superposition) condition.

Proof

$$v(t) = \text{Re}\{g(t)e^{j\omega_c t}\} = \tfrac{1}{2}g(t)e^{j\omega_c t} + \tfrac{1}{2}g^*(t)e^{-j\omega_c t}$$

Thus

$$V(f) = \mathcal{F}[v(t)] = \tfrac{1}{2}\mathcal{F}[g(t)e^{j\omega_c t}] + \tfrac{1}{2}\mathcal{F}[g^*(t)e^{-j\omega_c t}] \tag{4-24}$$

Using $\mathcal{F}[g^*(t)] = G^*(-f)$ from Table 2-1 and the frequency translation property of Fourier transforms from Table 2-1, this equation becomes

$$V(f) = \tfrac{1}{2}[G(f - f_c) + G^*(-(f + f_c))] \tag{4-25}$$

which reduces to (4-22).

Corresponding Amplitude and Phase Modulation			
$R(t)$	$\theta(t)$	Linearity	Remarks
$\lvert 1 + m(t)\rvert$	$\begin{cases} 0, & m(t) > -1 \\ 180°, & m(t) < -1 \end{cases}$	L[b]	$m(t) > -1$ required for envelope detection.
$\lvert m(t)\rvert$	$\begin{cases} 0, & m(t) > 0 \\ 180°, & m(t) < 0 \end{cases}$	L	Coherent detection required.
1	$D_p m(t)$	NL	D_p is the phase deviation constant (radian/volts).
1	$D_f \int_{-\infty}^{t} m(\sigma)d\sigma$	NL	D_f is the frequency deviation constant (radian/volt-sec).
$\sqrt{[m(t)]^2 + [\hat{m}(t)]^2}$	$\tan^{-1}[\pm \hat{m}(t)/m(t)]$	L	Coherent detection required.
$e^{\mp D_p \hat{m}(t)}$	$D_p m(t)$	NL	
$e^{\mp D_f \int_{-\infty}^{t} \hat{m}(\sigma)d\sigma}$	$D_f \int_{-\infty}^{t} m(\sigma)d\sigma$	NL	
$1 + m(t)$	$\pm \hat{\ln}[1 + m(t)]$	NL	$m(t) > -1$ is required so that the ln $(\cdot)$ will have a real value.
$\sqrt{1 + m(t)}$	$\pm \frac{1}{2}\hat{\ln}[1 + m(t)]$	NL	$m(t) > -1$ is required so that the ln $(\cdot)$ will have a real value.
$\sqrt{m_1^2(t) + m_2^2(t)}$	$\tan^{-1}[m_2(t)/m_1(t)]$	L	Used in NTSC color TV; requires coherent detection.

The PSD for $v(t)$ is obtained by first evaluating the autocorrelation for $v(t)$.

$$R_v(\tau) = \langle v(t)v(t + \tau)\rangle = \langle \mathrm{Re}\{g(t)e^{j\omega_c t}\}\mathrm{Re}\{g(t + \tau)e^{j\omega_c(t+\tau)}\}\rangle.$$

Using the identity (see Problem 2-42),

$$\mathrm{Re}(c_2)\ \mathrm{Re}(c_1) = \tfrac{1}{2}\ \mathrm{Re}(c_2^* c_1) + \tfrac{1}{2}\ \mathrm{Re}(c_2 c_1)$$

where $c_2 = g(t)e^{j\omega_c t}$ and $c_1 = g(t + \tau)e^{j\omega_c(t+\tau)}$, we get

$$R_v(\tau) = \langle \tfrac{1}{2}\ \mathrm{Re}\{g^*(t)g(t + \tau)e^{-j\omega_c t}e^{j\omega_c(t+\tau)}\}\rangle + \langle \tfrac{1}{2}\ \mathrm{Re}\ \{g(t)g(t + \tau)e^{j\omega_c t}e^{j\omega_c(t+\tau)}\}\rangle$$

Realizing that $\langle\ \rangle$ and Re{ } are both linear operators, the order of the operators may be exchanged without affecting the result, and the autocorrelation becomes

$$R_v(\tau) = \tfrac{1}{2}\,\mathrm{Re}\{\langle g^*(t)g(t+\tau)e^{j\omega_c\tau}\rangle\} + \tfrac{1}{2}\,\mathrm{Re}\{\langle g(t)g(t+\tau)e^{j2\omega_c t}e^{j\omega_c\tau}\rangle\}$$

or

$$R_v(\tau) = \tfrac{1}{2}\,\mathrm{Re}\{\langle g^*(t)g(t+\tau)\rangle e^{j\omega_c\tau}\} + \tfrac{1}{2}\,\mathrm{Re}\{\langle g(t)g(t+\tau)e^{j2\omega_c t}\rangle e^{j\omega_c\tau}\}$$

But $\langle g^*(t)g(t+\tau)\rangle = R_g(\tau)$. The second term on the right is negligible because $e^{j2\omega_c t} = \cos 2\omega_c t + j \sin 2\omega_c t$ oscillates much faster than variations in $g(t)g(t+\tau)$. In other words, f_c is much larger than the frequencies in $g(t)$, so the integral is negligible. This is an application of the Riemann-Lebesque lemma from integral calculus [Olmsted, 1961]. Thus, the autocorrelation reduces to

$$R_v(\tau) = \tfrac{1}{2}\,\mathrm{Re}\{R_g(\tau)e^{j\omega_c\tau}\} \tag{4-26}$$

The PSD is obtained by taking the Fourier transform of (4-26) (i.e., applying the Wiener–Khintchine theorem). Note that (4-26) has the same mathematical form as (4-21) when t is replaced by τ, so the Fourier transform has the same form as (4-22). Thus,

$$\mathcal{P}_v(f) = \mathcal{F}[R_v(\tau)] = \tfrac{1}{4}[\mathcal{P}_g(f - f_c) + \mathcal{P}_g^*(-f - f_c)]$$

But $\mathcal{P}_g^*(f) = \mathcal{P}_g(f)$ since the PSD is a real function. Thus, the PSD is given by (4-23).

Evaluation of Power

Theorem. The *total average normalized power* of a bandpass waveform, $v(t)$, is

$$P_v = \int_{-\infty}^{\infty} \mathcal{P}_v(f)df = R_v(0) = \tfrac{1}{2}\langle |g(t)|^2\rangle \tag{4-27}$$

where "normalized" implies that the load is equivalent to one ohm.

Proof. Substituting $v(t)$ into (2-67), we get

$$P_v = \langle v^2(t)\rangle = \int_{-\infty}^{\infty} \mathcal{P}_v(f)\,df$$

But $R_v(\tau) = \mathcal{F}^{-1}[\mathcal{P}_v(f)] = \int_{-\infty}^{\infty} \mathcal{P}_v(f)e^{j2\pi f\tau}\,df$, so

$$R_v(0) = \int_{-\infty}^{\infty} \mathcal{P}_v(f)\,df$$

Also, from (4-26)

$$R_v(0) = \tfrac{1}{2}\,\mathrm{Re}\{R_g(0)\} = \tfrac{1}{2}\,\mathrm{Re}\{\langle g^*(t)g(t+0)\rangle\}$$

or

$$R_v(0) = \tfrac{1}{2}\,\mathrm{Re}\{\langle |g(t)|^2\rangle\}$$

But $|g(t)|$ is always real, so

$$R_v(0) = \tfrac{1}{2}\langle |g(t)|^2 \rangle$$

Another type of power rating called the *peak envelope power* (PEP), is useful for transmitter specifications.

Definition. The *peak envelope power* (PEP) is the average power that would be obtained if $|g(t)|$ were to be held constant at its peak value.

This is equivalent to evaluating the average power in an unmodulated RF sinusoid that has a peak value of $A_p = \max[v(t)]$, as is readily seen from Fig. 4-32.)

Theorem. The *normalized PEP* is given by

$$P_{\text{PEP}} = \tfrac{1}{2}[\max|g(t)|]^2 \tag{4-28}$$

A proof of this theorem follows by applying the definition to (4-27). As described later in this chapter and in Chapter 5, the PEP is useful for specifying the power capability of AM, SSB, and television transmitters.

EXAMPLE 4-1 Amplitude-Modulated Signal

Evaluate the magnitude spectrum for an amplitude-modulated (AM) signal. From Table 4-1, the complex envelope of an AM signal is

$$g(t) = 1 + m(t)$$

so that the spectrum of the complex envelope is

$$G(f) = \delta(f) + M(f) \tag{4-29}$$

Using (4-19), the AM signal waveform is

$$s(t) = [1 + m(t)]\cos \omega_c t \tag{4-30}$$

and, using (4-22), the AM spectrum is

$$S(f) = \tfrac{1}{2}[\delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c)] \tag{4-31}$$

where, since $m(t)$ is real, $M^*(f) = M(-f)$, and $\delta(f) = \delta(-f)$ (the delta function was defined to be even) were used. Suppose that the magnitude spectrum of the modulation happens to be a triangular function, as shown in Fig. 4-4a. This spectrum might arise from an analog audio source where the bass frequencies are emphasized. The resulting AM spectrum, using (4-31), is shown in Fig. 4-4b. Note that since $G(f - f_c)$ and $G^*(-f - f_c)$ do not overlap, the magnitude spectrum is

$$|S(f)| = \begin{cases} \tfrac{1}{2}\delta(f - f_c) + \tfrac{1}{2}|M(f - f_c)|, & f > 0 \\ \tfrac{1}{2}\delta(f - f_c) + \tfrac{1}{2}|M(-f - f_c)|, & f < 0 \end{cases} \tag{4-32}$$

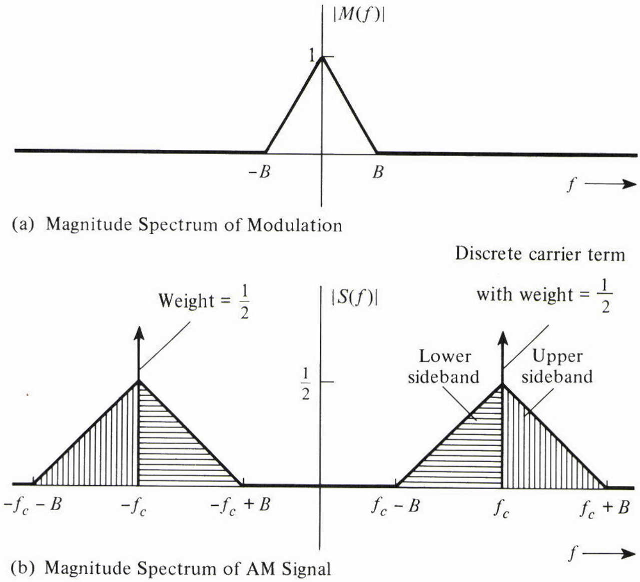

FIGURE 4-4 Spectrum of an AM signal.

The "1" in $g(t) = 1 + m(t)$ causes delta functions to occur in the spectrum at $f = \pm f_c$, where f_c is the assigned carrier frequency. Using (4-27), the total average signal power is

$$P_s = \tfrac{1}{2}\langle|1 + m(t)|^2\rangle = \tfrac{1}{2}\langle 1 + 2m(t) + m^2(t)\rangle$$

$$= \tfrac{1}{2}[1 + 2\langle m(t)\rangle + \langle m^2(t)\rangle]$$

Assuming that the dc value of the modulation is zero, as shown in Fig. 4-2, the average signal power becomes

$$P_s = \tfrac{1}{2}[1 + P_m] \tag{4-33}$$

where $P_m = \langle m^2(t)\rangle$ is the power in the modulation. ■

Digital Computer Simulation

Often communication systems are too complicated to analyze completely by using only analytical techniques, or a complete analytical approach would be too costly in terms of personnel and time. In these cases, *digital computer simulation* is used to provide a practical approach to system design and analysis. The complex envelope technique and sampling techniques are usually the basis of the computer algorithms. Using the complex envelope:

- Modulated signals can be represented (see Table 4-1).
- Noise can be represented (see Sec. 6-7).

- Filters can be simulated [see (4-9), (4-10), and Sec. 4-3].
- Detectors can be simulated (see Sec. 4-3).

Nonlinearities can be represented using time domain algorithms (see Sec. 4-3). The discrete Fourier transform (DFT) can be used for spectral analysis. Signal-to-noise ratios can be evaluated. Monte Carlo techniques can be used to estimate the probability of bit error for digital systems.

Several large-scale computer programs have been developed that simulate communication systems. Some of these programs are described in a special issue of the *IEEE Journal on Selected Areas in Communications*, vol. SAC-2, January 1984. One of these simulation programs, SYSTID, (system time domain) will now be studied in some detail [Fashano and Strodtbeck, 1984]. SYSTID has been in a continuous state of development and improvement by the Hughes Aircraft Company since 1967. Its development was supported under a National Aeronautics and Space Administration (NASA) contract during the early 1970s.

SYSTID uses the following techniques:

- Partitioning the communication system into basic functional blocks and processing the signal sequentially from the input to the output.
- Using complex envelope representation for modulated signals, noise, and filters.
- Using discrete time representation (sampling) of continuous time signals and systems.

The block diagram approach allows complicated communication systems to be built up from a set of interconnected functional blocks. The blocks are available from the SYSTID model library (see Table 4-2) or by calling user-defined models that are easy to program. The SYSTID package is written in FORTRAN code, and consists of a precompiler that takes SYSTID statements and generates the FORTRAN source code that simulates a particular communication system. For example, the SYSTID syntax for implementing a block in the simulation is

INPUT > MODEL NAME (PAR1, . . . PARn, . . .) > OUTPUT

where MODEL NAME is the name of the model selected from the SYSTID library (or one defined by the user), PARn are pass parameters for the model, INPUT is the name assigned to the input node of the block that is being simulated, and OUTPUT is the output node name. Blocks are connected together by using a common node name for the output of a block that connects to the input of another block.

The communication system is simulated by executing the machine code that results from the compiled FORTRAN source code after it has been linked to the SYSTID library routines. The program output consists of data that are samples of the waveform (as a function of time) at each node of interest. A companion analysis program called RIP is used to look at the SYSTID output data and present a graphic output of the user-requested waveform properties. For example, the user may request plots of the waveform at any node, the PSD, the

TABLE 4-2 SYSTID Model Library

Signal Generators	Filters	Miscellaneous	Estimators
White Gaussian noise	Butterworth	Limiters	Power spectra
Arbitrary PDF sources	Chebychev	TWT	Probability of bit error (MPSK, QASK)
Pulse gen.	Bessel	Arbitrary nonlinearities	Noise power ratio
Transcendental functions	Butterworth–Thompson	TDA	Signal distortion
Square wave	Elliptic	Up/down converters	Signal to noise
Arbitrary tables	Quasi-elliptic	Differentiator	Noise bandwidth
PN seq. gen.	Lead lag	Integration	Delay
Random bit stream gen.	Integrate and dump	Time delays	Correlation
PN bit stream gen.	Quadratic	Phase shifters	Statistics
MSK PN signal gen.	Arbitrary poles and zeros	Latches and gates	Eye pattern analyzer
Raised cos PN signal gen.	Transversal	Threshold detectors	Intelligible crosstalk
QPSK PN signal gen.	Zonal (ideal)	Power meter	**Source Encoders**
FM noise loaded baseband signal gen.	Measured response	Average meter	Analog-to-digital
Biphase L PN gen.	Functional form	Rotary joint multipath	Digital-to-analog
ENB signal gen.	Adaptive transversal equalizer	Recall file	Multilevel PCM
Impulse noise	**Mods and demods**	Saver/loader	Orthogonal
Continuous wave	Amplitude		Fourier
	Frequency		Haar
	Phase		Hadamard
	Delta modulator		Ordered Hadamard
	FSK		Digital interleaving and deinterleaving
	PSK		Convolutional
	QASK, MPSK		
	Digital echo modulation		
	Zero crossing detector		

Source: Michael Fashano and Andrew L. Strodtbeck, "Communication System Simulation and Analysis with SYSTID," *IEEE Journal on Selected Areas in Communications*, vol. SAC-2, Janaury 1984, p. 11. © 1984 IEEE.

autocorrelation function, the eye pattern, etc. An example of a SYSTID simulation for a simple communication system and the associated RIP graphic output is shown in Fig. 4-5.

The SYSTID package can be used to simulate very complicated communication systems. Some typical applications include bit error rate sensitivity, adjacent and cochannel interference, multicarrier frequency division multiple access (FDMA) frequency planning (see Chapter 5 for a description of FDMA), modulation and coding system performance, analog system performance (S/N and distortion), signal detection, adaptive signal processing, device modeling and verification of channel specifications.

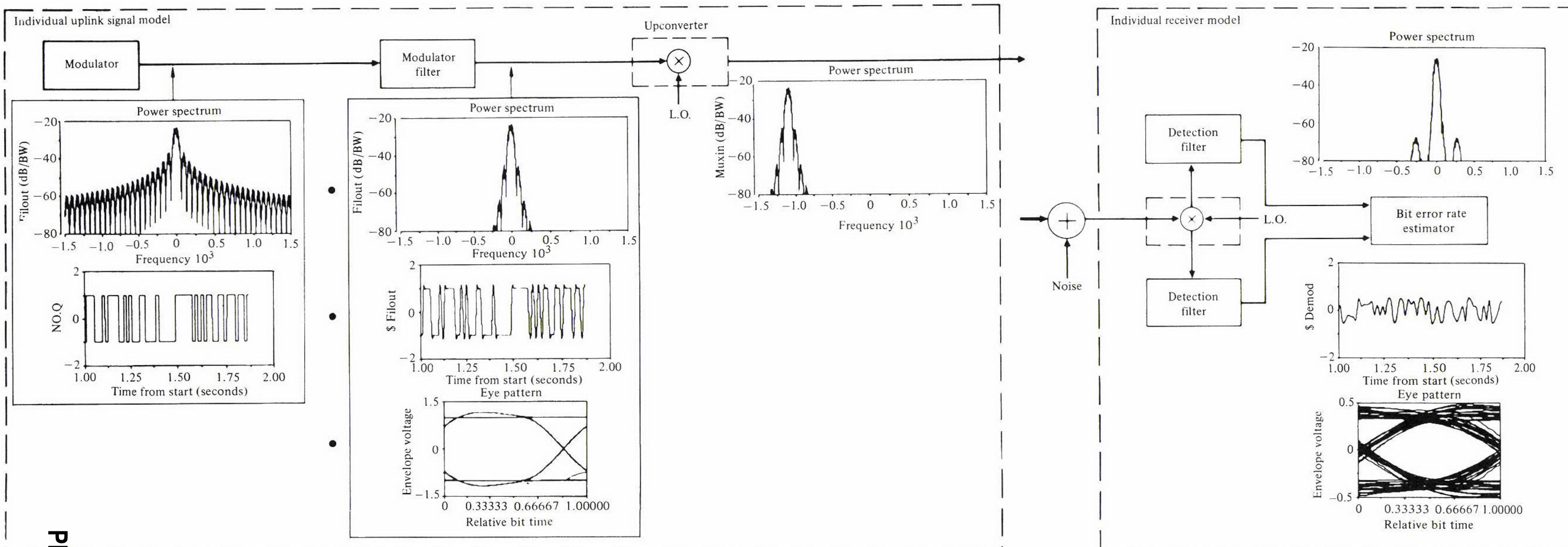

FIGURE 4-5 Transmitter and receiver simulation using SYSTID. [From Michael Fashano and Andrew L. Strodtbeck, "Communication System Simulation and Analysis with SYSTID," *IEEE Journal on Selected Areas in Communications*, vol. SAC-2, Jan. 1984, p. 17. © 1984 IEEE.]

From the preceding discussion, it is realized that a study of communication system component blocks is needed before an overall study of communication systems (as presented in Chapter 5) is attempted. This study of the basic building block components is presented in the next section.

4-3 COMPONENTS OF COMMUNICATION SYSTEMS

Communication systems consist of component blocks such as amplifiers, filters, limiters, oscillators, mixers, frequency multipliers, phase-locked loops, detectors, and other analog and digital circuits. In most circuit and electronic courses, a great deal of time is spent on circuits that are linear and time invariant since they are relatively easy to analyze and form the basis for understanding other types of circuits. However, it is realized that all practical circuits are nonlinear, and we need to examine the distortion products that are produced by nonlinear effects. Also, as we shall see, nonlinear and time-varying circuits are often needed in communication systems to produce signals with new frequency components.

Filters

As we know, filters are devices that take an input waveshape and modify the frequency spectrum to produce the output waveshape. Filters may be classified in several ways. One is by the type of construction used, such as *LC* elements or quartz crystal elements. Another is by the type of transfer function that is realized, such as the Butterworth or Chebyshev response (defined subsequently). These two topics of construction types and transfer function characteristics are discussed in this section.

Filters use energy storage elements to obtain frequency discrimination. In any physical filter the energy storage elements are imperfect. For example, a physical inductor has some series resistance as well as inductance, and a physical capacitor has some shunt (leakage) resistance as well as capacitance. A measure of the goodness of a circuit element, or, moreover, a measure of goodness for the whole circuit is given by the quality factor Q.

$$Q = \frac{2\pi(\text{maximum energy stored during one cycle})}{(\text{energy dissipated per cycle})} \tag{4-34}$$

Of course, a larger value for Q corresponds to a more perfect storage element. That is, a perfect L or C element would have infinite Q. For an *RLC* series or parallel resonant circuit, the Q is

$$Q = \frac{f_o}{B} \tag{4-35}$$