Appendix

Proof of (12): We use (10), (11), the Schwarz inequality, and the unitarity of the spreading function

$$\begin{aligned}
&\|L_{GH} - L_G L_H\|^2 = \|S_{GH} - S_G ** S_H\|^2 \\
&= \int_{-2\tau_0}^{2\tau_0}\int_{-2\nu_0}^{2\nu_0} \left| \int_{\tau'}\int_{\nu'} S_G(\tau', \nu') S_H(\tau - \tau', \nu - \nu') \right. \\
&\quad \left. \cdot \left(e^{-j2\pi[\tau\nu'(1/2-\alpha) - \tau'\nu(1/2+\alpha) + 2\tau'\nu'\alpha]} - 1\right) d\nu' d\tau' \right|^2 d\nu d\tau \\
&\le \int_{-2\tau_0}^{2\tau_0}\int_{-2\nu_0}^{2\nu_0} \left(\int_{\tau_1}\int_{\nu_1} |S_G(\tau_1, \nu_1)|^2 d\nu_1\, d\tau_1 \right) \\
&\quad \cdot \left(4 \int_{\tau_2}\int_{\nu_2} |S_H(\tau_2, \nu_2)|^2 \sin^2(\pi[\tau\nu_2(1/2 - \alpha) \right. \\
&\qquad \left. - \tau_2\nu(1/2 + \alpha) + 2\tau_2\nu_2\alpha]) d\nu_2 d\tau_2 \right) d\nu d\tau \\
&< 64\tau_0\nu_0 \|\mathbf{G}\|^2 \max_{|\tau_i| < \tau_0, |\nu_i| < \nu_0} \\
&\quad \times \left\{ \sin^2(\pi[\tau_1\nu_2(1/2 - \alpha) - \tau_2\nu_1(1/2 + \alpha) + 2\tau_2\nu_2\alpha]) \right\} \\
&\quad \cdot \left(\int_\tau \int_\nu |S_H(\tau, \nu)|^2 d\tau\, d\nu \right) \\
&= 64\tau_0\nu_0 \sin^2(\pi\tau_0\nu_0(1 + 2|\alpha|)) \|\mathbf{G}\|_{HS}^2 \|\mathbf{H}\|_{HS}^2.
\end{aligned}$$

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A New Volterra Predistorter Based on the Indirect Learning Architecture

Changsoo Eun and Edward J. Powers

***Abstract*—In this correspondence, we present a new Volterra-based predistorter, which utilizes the indirect learning architecture to circumvent a classical problem associated with predistorters, namely that the desired output is not known in advance**

I. Introduction

Nonlinear compensation techniques are becoming increasingly important to improve the performance of telecommunication channels by compensating for channel nonlinearities. Nonlinear compensators can be classified into two categories: equalizers and predistorters, located after and before the nonlinear channel, respectively.

The pth-order inverse method [1] based on the Volterra series model was applied to predistorter design [2] for nonlinear telecommunication channels. However, the design of the pth-order inverse system is very complicated and must be based on a known Volterra series model of the nonlinear channel. Moreover, as the order of nonlinearity grows higher, so does the design complexity of the pth-order inverse method.

To the best of the authors' knowledge, nonlinear predistorter design using a Volterra series model, other than the pth-order inverse method, has not been attempted. This is due to the fact that, beforehand, the desired output values of the predistorter are unknown. To overcome this difficulty, we utilize the indirect learning architecture and the recursive least square (RLS) algorithm. Specifically, we propose in this correspondence an indirect Volterra series model predistorter which is independent of a specific nonlinear model for the system to be compensated, as opposed to the pth-order inverse method which first requires a Volterra series model of the system or channel to be compensated. We use the phrase *indirect Volterra* to distinguish our new predistorter from the more common pth-*order inverse* approach, and in recognition that our approach rests upon utilizing the *indirect* learning architecture. Both 16–phase shift keying (PSK) and 16-quadrature amplitude modulation (QAM) will be used to demonstrate the efficacy of the new approach.

II. Volterra Model and the pth-Order Inverse Method

In discrete time, a third-order Volterra series for a causal, finite-memory system becomes

$$\begin{aligned}
y[n] &= \sum_{k=0}^{N} h_k^{(1)} x[n-k] + \sum_{k=0}^{N}\sum_{l=0}^{N} h_{k,l}^{(2)} x[n-k]x[n-l] \\
&\quad + \sum_{k=0}^{N}\sum_{l=0}^{N}\sum_{m=0}^{N} h_{k,l,m}^{(3)} x[n-k]x[n-l]x[n-m] \\
&\quad + e[n] \qquad (1)
\end{aligned}$$

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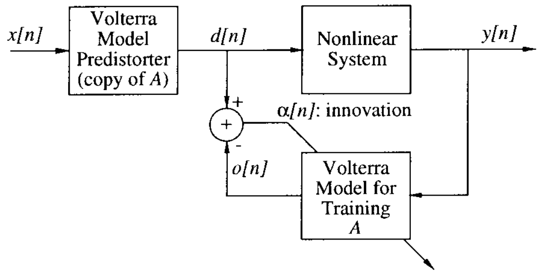


Fig. 1. The training architecture of the Volterra series model predistorter.

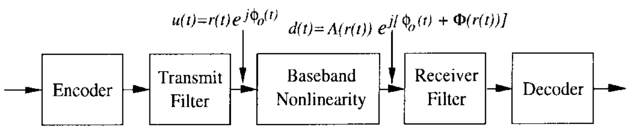


Fig. 2. The satellite communication channel model reduced to base band.

where N is the discrete system memory length; $x[n]$ and $y[n]$ the input and output, respectively; and $h_k^{(1)}$, $h_{k,l}^{(2)}$, and $h_{k,l,m}^{(3)}$ are the discrete time domain Volterra kernels of order 1, 2, 3, respectively.

The pth-order inverse method, based on the Volterra series model, was proposed by Schetzen in [1]. This method connects another Volterra system in series with the nonlinear system to be compensated such that the overall Volterra kernels of order greater than one and less than or equal to p are zero. However, the kernels of the overall system for orders higher than p are not zero, even though these higher order nonlinearities may not be present in the original system. They may have an undesirable effect on the nonlinear compensation depending on the input signal level.

III. The Direct Volterra Series Model Approach

To circumvent the problem of not knowing beforehand the desired output of a predistorter, we utilize the *indirect learning architecture* [3] indicated in Fig. 1. This algorithm uses two identical Volterra models for the predistorter and training. As the innovation $\alpha[n]$ approaches zero, the overall output of the system $y[n]$ approaches the system input $x[n]$, i.e., the desired output of the overall system, since the inputs to the identical networks are equal. After the training session, the training network is removed. We will call this approach the *indirect* Volterra series model approach since we use an independent Volterra series model instead of the pth-order inverse system as a predistorter. Additional details follow.

The third-order Volterra series model of (1) is expressed in a matrix form as

$$d[n] = \mathbf{h}\mathbf{x}^T[n] \tag{2}$$

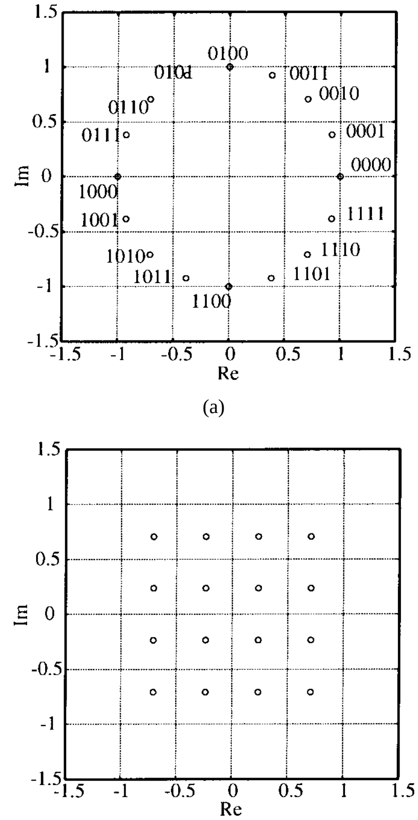
where $d[n]$ is the output of the Volterra series model predistorter, the superscript T denotes the transpose of the matrix, $\mathbf{h}$ is the Volterra kernel vector, and $\mathbf{x}[n]$ is the input vector, which are defined by

$$\begin{aligned}\mathbf{h} = [&h_0^{(1)}, h_1^{(1)}, \cdots, h_{N-1}^{(1)}, h_{000}^{(3)}, h_{001}^{(3)}, h_{002}^{(3)} \\ &\cdots, h_{klm}^{(3)}, \cdots, h_{(N-1)(N-1)(N-1)}^{(3)}]\end{aligned} \tag{3}$$

$$\begin{aligned}\mathbf{x}[n] = [&x[n], x[n-1], \cdots, x[n-N+1], |x[n]|^2 x[n] \\ &|x[n]|^2 x[n-1], |x[n]|^2 x[n-2], \\ &\cdots, |x[n-N+1]|^2 x[n-N+1]].\end{aligned} \tag{4}$$

Here, n is the current sample time and N is the system memory length in the discrete time form. The overall system output $y[n]$ is given by

$$y[n] = \sigma(\mathbf{d}[n]) \tag{5}$$



(a)

(b)

Fig. 3. Constellations of the original (a) 16-PSK and (b) 16-QAM signals. A simple example of data assignment is shown for the 16-PSK constellation.

where σ is a nonlinear system function with memory and the vector $\mathbf{d}[n]$ is given by

$$\mathbf{d}[n] = [d[n], d[n-1], \cdots, d[n-M+1]]. \tag{6}$$

Here, M is the memory duration of the nonlinear system.

The output $o[n]$ of the training Volterra series model is given by

$$o[n] = \mathbf{h}\mathbf{y}^T[n] \tag{7}$$

where $\mathbf{y}[n]$ is the vector of the sampled output of the overall system defined by

$$\begin{aligned}\mathbf{y}[n] = [&y[n], y[n-1], \cdots, y[n-N+1], |y[n]|^2 y[n] \\ &|y[n]|^2 y[n-1], |y[n]|^2 y[n-2] \\ &\cdots, |y[n-N+1]|^2 y[n-N+1]].\end{aligned} \tag{8}$$

If the Volterra series models satisfy the following conditions

$$\text{if } \mathbf{x}[n] \neq \mathbf{y}[n], \quad \text{then } d[n] \neq o[n]$$

and

$$\text{if } \mathbf{x}[n] = \mathbf{y}[n], \quad \text{then } d[n] = o[n] \tag{9}$$

then, as $\alpha[n] = d[n] - o[n]$ approaches zero, $\mathbf{y}[n]$ approaches $\mathbf{x}[n]$, thus so does $y[n]$ to $x[n]$.

The kernel update of the training Volterra series system follows the modified RLS algorithm. The kernel vector at the sample time n is updated by

$$\hat{\mathbf{h}}^{(n)} = \hat{\mathbf{h}}^{(n-1)} + \mathbf{k}[n]\alpha^*[n] \tag{10}$$

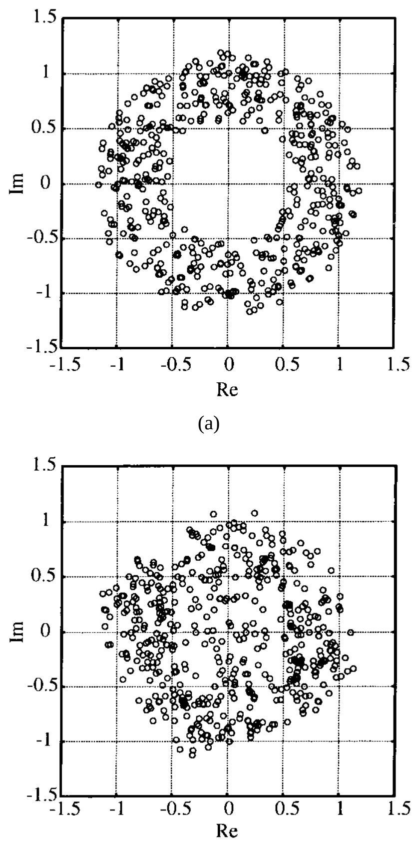


Fig. 4. The constellation of (a) the 16-PSK signal and (b) the 16-QAM signal distorted by the linear filters and TWT of the satellite communication channel. The maximum amplitude of the input signal is one.

where $\alpha[n]$ is called the innovation that represents the information contained in the current desired output value which cannot be predicted by the previous kernel vector and is defined by

$$\alpha[n] = d[n] - \hat{\mathbf{h}}^{(n-1)}\mathbf{y}^T[n] \tag{11}$$

$$= \hat{\mathbf{h}}^{(n-1)}(\mathbf{x}[n] - \mathbf{y}[n])^T \tag{12}$$

and $\mathbf{k}[n]$ is the time-varying gain vector defined by

$$\mathbf{k}[n] = \frac{\lambda^{-1}\mathbf{P}[n-1]\mathbf{y}^T[n]}{1 + \lambda^{-1}\mathbf{y}^*[n]\mathbf{P}[n-1]\mathbf{y}^T[n]}. \tag{13}$$

The matrix $\mathbf{P}[n]$ is updated as follows:

$$\mathbf{P}[n] = \lambda^{-1}\mathbf{P}[n-1] - \lambda^{-1}\mathbf{k}[n]\mathbf{y}^*[n]\mathbf{P}[n-1]. \tag{14}$$

Note that the output $\mathbf{y}[n]$ of the overall system is used as the input to the training Volterra series model. The contents of the training Volterra series model are copied into those of the Volterra series model predistorter.

IV. Satellite Telecommunication Channel

In Fig. 2, we show a simplified base band satellite telecommunication channel. In such channels, the data or symbols to be transmitted are passed through a pulse shaping filter for effective bandwidth use. At the satellite, the weak signal is amplified by a microwave power amplifier, usually a traveling wave tube (TWT) or a solid-state amplifier. The amplifier is normally operated near saturation to maximize power efficiency as the power available on board the satellite is limited. Amplifier saturation introduces a nonlinear dependence of the output amplitude and phase on input amplitude levels thereby resulting in AM/AM and AM/PM conversion, respectively. The amplitude A and the phase shift Φ of the output signal of the TWT are modeled by [4] as follows:

$$A[r(t)] = \frac{\alpha_a r(t)}{1 + \beta_a r^2(t)} \tag{15}$$

$$\Phi[r(t)] = \frac{\alpha_p r^2(t)}{1 + \beta_p r^2(t)} \tag{16}$$

where α_a, β_a, α_p, β_p are constants and $r(t)$ is the normalized input.

At the receiver a filter rejects noise and unwanted signals, and then decodes and detects. The demodulated signal $d(t)$ is given by

$$d(t) = A[r(t)]e^{j\{\phi_0(t)+\Phi[r(t)]\}} \tag{17}$$

where the symbol input to the channel is $u(t) = r(t)e^{j[\phi_0(t)]}$.

Even though the TWT may be regarded as a zero-memory nonlinear system, the overall telecommunication channel shown in Fig. 2 can be considered as a nonlinear system with memory.

V. Numerical Experiment Results

To compare the performance of the *indirect* and pth-order predistorters, we will use the normalized mean-squared error (NMSE) defined by

$$\text{NMSE} = \frac{\sum_{k=1}^{K} |x_k - o_k|^2}{\sum_{k=1}^{K} |x_k|^2} \tag{18}$$

where K is the number of total test data, x_k is the desired detector output of kth test data, and o_k is the kth compensated output. Note that in the compensated system, the desired output is the input to the overall system.

In this example, we use the base-band satellite communication channel model depicted in Fig. 2. We note that the linear filters, because of their memory characteristics, cause intersymbol interference (ISI). The linear filter coefficients used in this example are given by

$$\mathbf{h} = [0.8,\ 0.1] \tag{19}$$

$$\mathbf{g} = [0.9,\ 0.2,\ 0.1] \tag{20}$$

where $\mathbf{h}$ and $\mathbf{g}$ are the transmit and receiver filter coefficient vectors, respectively. These numbers are chosen to represent general intersymbol interference in which the effect of previous symbols decreases with time.

The nonlinearity caused by the TWT can be represented by (15), (16) with the typical values of constants $\alpha_a = 2.0$, $\beta_a = 1.0$, $\alpha_p = \pi/3$, $\beta_p = 1.0$ [5]. In Fig. 3 we show 16-PSK and 16-QAM constellations to be transmitted, and in Fig. 4 we present the distorted constellations after transmission through the nonlinear channel. Without compensation, high symbol error rate would seriously degrade system performance.

For comparison purposes, we first modeled the system with a third-order Volterra series model in order to apply the pth-order inverse method. The memory span of the model was assumed to be three, since the memory of the ISI is three—one for the transmitter filter and two for the receiver filter. Then the pth-order inverse was determined.

Unlike the pth-order inverse method, the indirect Volterra series model approach does not need a Volterra series model of the system to be compensated. Therefore, utilizing the linear filter coefficients

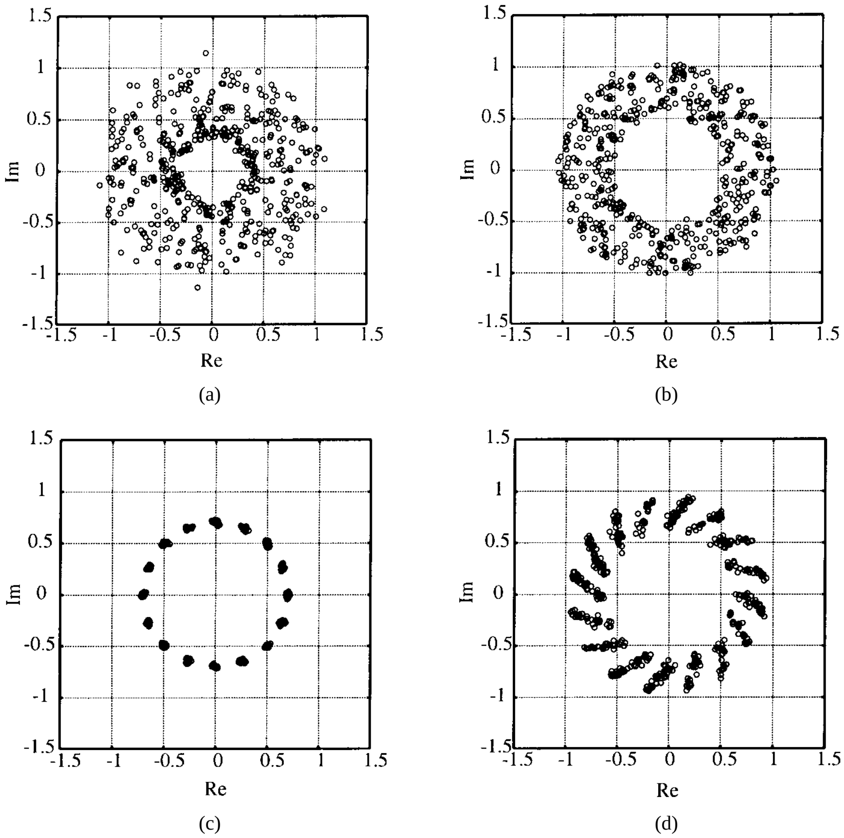


Fig. 5. 16-PSK constellations of signals: (a) predistorted by the direct Volterra series model predistorter; (b) predistorted by the pth-order inverse system; (c) compensated by the direct Volterra series model predistorter (NMSE $=$ 0.0007); and (d) compensated by the pth-order inverse system (NMSE $=$ 0.088) for the satellite communication channel. SNR $= \infty$.

and the parameters of the TWT, which were previously given, we obtain, using the indirect learning architecture, a Volterra series model predistorter to compensate the undesired nonlinearity. In both cases we consider third-order models.

The channel reduces the amplitudes of the input signal as shown in Fig. 4. The predistorter should "amplify" the signal to compensate for the amplitude reduction. Therefore, if the level of the input to the predistorter is one, the level of the output from the predistorter becomes larger than one due to the amplification. If this amplified signal is fed to the communication channel, the input signal to the TWT may exceed the maximum input level that produces the maximum output power. Therefore, for the predistorter output to fit into the TWT input range for maximum output power, the maximum level of the input to the predistorter was reduced to 0.64. Therefore, the maximum level of the desired compensated output is 0.64. This can be seen in the compensated constellations in Figs. 5(c) and 6(c). This may be a drawback of the predistorter in this particular case, since the reduced signal amplitude means a lower output signal-to-noise ratio and shorter Euclidean distance between the symbols, which means a higher detection error probability.

In the following, we compare the performance of the indirect Volterra and pth-order inverse distorters using both 16-PSK and 16-QAM signals.

In Fig. 5, we show results obtained from the two predistorters for 500 16-PSK symbols. In Fig. 5(a) and (b), we show the constellations of 16-PSK signals predistorted by the indirect Volterra series model and the pth-order inverse predistorters, respectively. In Fig. 5(a), we see that TWT nonlinearity is compensated for by increasing the "gain" of the predistorter with the signal level. Here, what we mean by "gain" of a predistorter or a filter is the amplitude variation introduced by digital signal processing. The increasing gain of the predistorter results in nonuniform symbol distribution in which symbol density is densest at small amplitudes and becomes sparser with increasing amplitude. However, the pth-order inverse result of the predistorter shown in Fig. 5(b) does not show this compensation effect. In Fig. 5(c) and (d), we show the compensated output symbol constellation. The performance of the indirect Volterra series predistorter is quite good (NMSE $=$ 0.0007). On the other hand, the output of the channel compensated by the pth-order inverse predistorter shows that the compensation is not as effective (NMSE $=$ 0.088). Although some of the effects of clustering have been reduced, we note significant amplitude and phase distortion remain.

Next, we compare the two approaches using a 16-QAM signal. In Fig. 6, we show the predistorter output for the 16-QAM signal using both the indirect Volterra series model approach Fig. 6(a) and the pth-order inverse method Fig. 6(b). Examining Fig. 6(d) for the pth-order inverse we note that, although the clustering has been reduced, both significant amplitude and phase distortion remain, especially for the larger amplitudes. On the other hand, the performance of the indirect Volterra predistorter shown in Fig. 6(c) is quite good as manifested by its NMSE of 8.0×10^{-4}, compared to 0.17 for the pth-order predistorter.

The relatively poor performance of the pth-order predistorter, in both the 16-PSK and 16-QAM cases, may very well be due to the generation of higher order $(>p)$ nonlinearities by the pth-order inverse predistorter.

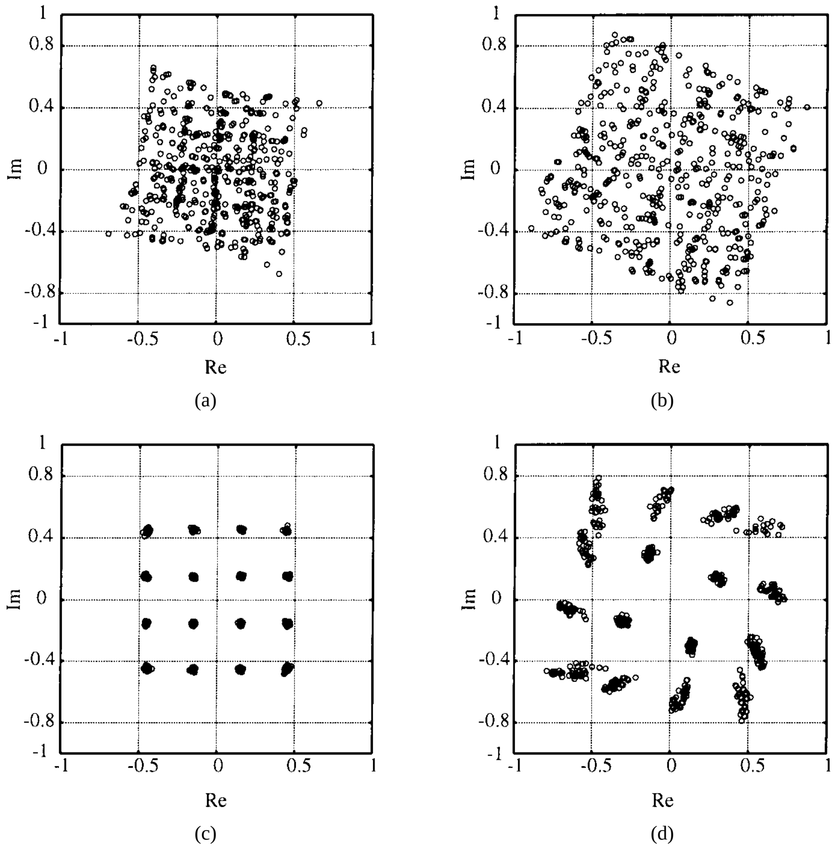


Fig. 6. The 16-QAM constellations of signals: (a) predistorted by the direct Volterra series model predistorter; (b) predistorted by the pth-order inverse system; (c) compensated by the direct Volterra series model predistorter (NMSE $=$ 0.0008); and (d) compensated by the pth-order inverse system (NMSE $=$ 0.17) for the satellite communication channel. SNR $= \infty$.

VI. Discussion

The results show that, as well as having a much simpler design procedure, the indirect Volterra series model approach has other advantages over the pth-order inverse method. The indirect Volterra series model approach does not need a specific nonlinear model, thus eliminating the need to first determine a pth-order Volterra model of the nonlinear channel. Furthermore, with the indirect Volterra approach, no pth-order inversion is required. The generation of higher order (greater than p) nonlinearities is reduced, since the design procedure is an optimization (in a least squares sense) instead of a system inversion.

The indirect Volterra series model approach is preferable if the order of the nonlinearity is low (say, third or possibly fifth order). If the order of the nonlinearity is higher (greater than, say, third or fifth order), then both the indirect Volterra series model approach and the pth-order inverse method are usually not practical because of the increased complexity. For example, the number of unknown kernel coefficients increases rapidly. In such cases, neural-network-based predistorters, designed using the indirect learning architecture, appear to be an appropriate alternative, an approach we have investigated and will report upon elsewhere.

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